XENON POISONING AND SPATIAL OSCILLATIONS IN A NUCLEAR REACTOR : A FEASIBILITY STUDY FOR EXPERT SYSTEM BASED OPTIMAL CONTROL

by

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CERTIFICATE

Certified that this work "Xenon Poisoning And Spatial Oscillations In A Nuclear Reactor : A Feasibility Study For Expert System Based Optimal Control" by D. Venkata Chalapathy has been carried out under our supervision and has been not submitted elsewhere for the award of a degree.

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ABSTRACT

The fission product poison Xenon-135 has been identified to ause two main problems in operation of a thermal nuclear power reactor. One is buildup of Xenon concentration following the mmediate shutdown of a reactor, which can correspond to several tens of dollars of negative reactivity. Unless proper preventive steps are taken, the reactor will lose the capability to restart after a few hours of shutdown and may remain in this state for 30-50 hours depending on the size and type of the reactor. The second problem is the Xenon induced spatial oscillations in the axial direction in the reactor, which can cause "hotspots". This problem assumes importance during load-follow operation. A suitable control means has to be adopted to eliminate these oscillations soon after their onset. This thesis addresses to these two problems and describes algorithmic as well as expert system based models to arrive at optimal control strategies.

The optimal shutdown control problem has ben solved using the Pontryagin's Maximum Principle. The concept of Xenon - Iodine Phase Plane has been used to develop the control strategy. An example of a point reactor model has been used to generate the numerical results for different steady state operating conditions which form the basis for knowledge base of the expert system

model. A conceptual model of the expert system, so developed, for shutdown control have been discussed.

The Xenon Induced Oscillations problem has ben formulated using lambda mode expansion method, and analytical solutions have been obtained using Pontryagin's Principle. The method has been applied to obtain control strategies for a case of pressurized water reactor. An expert system for the optimal control of spatial oscillations has also been proposed.

The expert system based models are found to offer advantages over conventional algorithmic methods due to their ability to incorporate heuristic knowledge and improved computational speed. Thus they are more suitable for real time applications.

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CHAPTER 1

INTRODUCTION

The phenomenon of excessive neutron absorption due to presence of certain fission products is known as *poisoning*. The most important fission product in a thermal nuclear power reactor contributing to poisoning is Xenon-135, whose thermal neutron absorption cross-section is 2.65 million barns. Poisoning-out occurs when a high flux thermal reactor is abruptly shut down leading to a rapid build-up of Xenon-135 within the core. If the concentration exceeds a prescribed maximum limit, the reactor cannot be restarted until the excess poison has decayed away naturally, which may take 30-50 hours. Spatial power oscillations are most likely to occur in large, high-power, non-boiling thermal reactors. A local perturbation in flux level of such a reactor due to any cause such as load following operation, can trigger a cyclic variation in Xenon concentration in various regions of the reactor volume. Since the neutron flux, and consequently the reactor power, depends on the Xenon profile, such Xenon oscillations produce the effect of a moving "hot-spot". Under certain conditions such oscillations may diverge causing severe damage to the structural materials, and is a serious safety concern. Suitable control means are therefore required to overcome the effects of poisoning-out and power oscillations.

Literature Survey And Motivation

Rosztoczy and Weaver [1] formulated the Xenon poisoning problem as one that can be solved by the use of Pontryagin's Maximum Principle [15]. They conclude that bang-bang control would be required for minimum Xenon build-up, and use this information to obtain optimal solution by a trial-and-error procedure. Roberts and Smith [2] defined optimality as the minimum time control program subject to a constraint on the maximum allowable Xenon level and applied the Maximum Principle to find time-optimal solutions for various steady state operating points. The methods of Dynamic Programming as developed by Bellman [16] were used by Ash [3] to generate shutdown flux program that would minimize the maximum Xenon following the completion of the flux program. The physics of Xenon-shutdown problem and methods of solution were discussed by Ash [4].

The analysis of time-optimal control of Xenon spatial oscillations has been extensively studied [4-10]. Canosa and Brooks [9] used one-group space-dependent kinetic equations and linearized theory to predict onset of oscillations. They also obtained explicit formulas for the effect of modal coupling of the fundamental and first harmonic with the infinite reactor modes. A control strategy based on control theory concepts and physics of the problem was developed by Christie and Poncelet [5]. The control is optimal in the sense that the control actions are minimized in number and are most effective in eliminating the oscillations. El-Bassioni and Poncelet [7] conclude that bang-bang control would be required for minimal time modal control strategy

to suppress Xenon oscillations in nuclear reactors. The control action depends on the exact knowledge of the state of the reactor which depends on the measured axial offset; the concept of the axial offset phase plane is introduced. Wiberg [6] used the maximum principle to get a feedback control law which was optimal in the sense of minimizing the mean square flux deviation from its reference as well as control energy. Ukai, *et al*, [8], investigated the control problem of the flux spatial distribution during a load-follow of a PWR as the generalized servomechanism problem. Chiang, *et al*, [10], formulated the time optimal control problem in terms of lambda mode [17-18] solutions to the static reactor balance equations. The spatial Xenon and Iodine imbalances have been used to form the axes of the control phase plane.

For control of the Xenon related problems, mathematical models incorporating the physical phenomena in all their complexity would be too cumbersome for treatment *via* conventional analytical methods. Hence, in the past, simplified mathematical models were used to obtain control strategies for minimization of Xenon level following immediate shutdown [1-4] , and for suppression of Xenon induced spatial power oscillations [5-10]. In order to bridge the gap between the control strategy developed using a simplified model and the control required for the actual physical system, the application of artificial intelligence (AI) techniques was suggested as an effective means [11]. Through the use of AI techniques, an entirely new philosophy of control is now possible. This is mainly because, AI based systems are able to incorporate heuristics and other operational knowledges of the physical system to be controlled into their programs. This ability

can be beneficially used to modify the output obtained from use of a simplified model to generate a more acceptable control output. In general, potential improvements in plant efficiency and reliability are often cited as reasons for developing and applying AI technologies, particularly expert systems, to control and operation of nuclear reactors [12-14].

Shinohara [11] discusses a general approach for the application of AI methods to the reactor control problems, considering only the Xenon shutdown problem. However it was not applied to any specific example and did not consider the problem of Xenon oscillations. Hence the aim of the present of the thesis was to develop an expert system based optimal control model for the shutdown problem and as well as the Xenon induced spatial power oscillations problem. A specific example of pressurized water reactor has been considered to develop the expert system based controls. Algorithmic approaches based on Pontryagin's principles of optimal processes have been described in detail for the two control problems and have been used to generate the control strategies for different operating conditions. These results of algorithmic approach along with certain heuristic rules form the basis for the knowledge base of the expert systems.

Thesis Organization

The present chapter introduces the Xenon related problems and sets the motivation behind the works carried out in the thesis. A brief review of various analytical and AI methods applied to Xenon

control is presented.

Chapter 2 is devoted to the optimal shutdown control problem. The concept of Xenon-Iodine Phase Plane (XIPP) representation is introduced which is used to define and generate time-optimal trajectories for various steady-state operating points.

A brief introduction to Artificial Intelligence and expert systems is given in Chapter 3. The development of an expert system for optimal shutdown control of nuclear reactors has been discussed.

Chapter-4 begins with an introduction to the control of spatial Xenon oscillations. The lambda mode expansion method is used to obtain a time optimal control strategy. An expert system is proposed for suppression of the oscillations.

Chapter-5 presents a summary of major findings in the present thesis and the scope for future research.

CHAPTER 2

OPTIMAL SHUTDOWN CONTROL

2.1. Introduction

This chapter is devoted to optimal shutdown control of a thermal nuclear power reactor. For arriving at the optimal control strategy, mainly two approaches, one based on Dynamic Programming technique [3] and the other based on Pontryagin's Maximum Principle [1,2] have been used. Amongst the two, Dynamic Programming based method requires more memory space and computational time. Hence, the Pontryagin's Maximum Principle has been used in this chapter to obtain the time optimal control strategy. The concept of the Xenon-Iodine Phase Plane (XIPP) is used to obtain the time optimal trajectories. These trajectories were determined for various steady state operating points of a point reactor model.

2.2. Nuclear Reactor Poisons

The core of a high flux thermal reactor consists of a variety of non-fuel elements which constitute the structural materials, fuel cladding, moderator etc. In addition, a large number of new fission product elements are being constantly produced, either directly (Fig 2.1), or through a decay chain (Fig 2.2), from the fission process. All these elements have the inherent property of absorbing neutrons to a lesser or a greater extent. Thus their presence leads to a reduced core neutron economy, which has a

profound effect on the equilibrium operation of the reactor. Any such undesirable neutron absorbing material present in the reactor core is called a *poison*. Reactor poisons are always not undesirable; Boron (Table 2.1) is a control poison used to neutralize excessive reactivity present in a reactor. In PHWRs, Boron is used as a backup protection for emergency reactor shutdown (scram).

Xenon-135 is the most important fission product poison by virtue of it's enormous absorption cross-section (Table 2.1). It is formed both directly through fissions and through β-decay of Iodine-135. It is removed both by neutron absorption forming inert Xe-136 and through the natural β-decay process forming Cesium-135, a harmless non-poison.

Of other fission products, Samarium-149 has the next highest thermal neutron absorption cross section (Table 2.1). It is formed through two successive β-decays of the fission product Neodymium-149 and removed through neutron absorption forming inert Sm-150 (Fig 2.2). However, the absorption cross section of Sm-149 is small compared to that of Xe-135. Therefore, optimal control problem depends primarily on Xe-135 concentration. In the theoretical considerations of reactor poisons to follow, the emphasis will be on Xe-135 alone. The usage "Xenon" and "Iodine" refers to Xe-135 and I-135 respectively, unless otherwise specified.

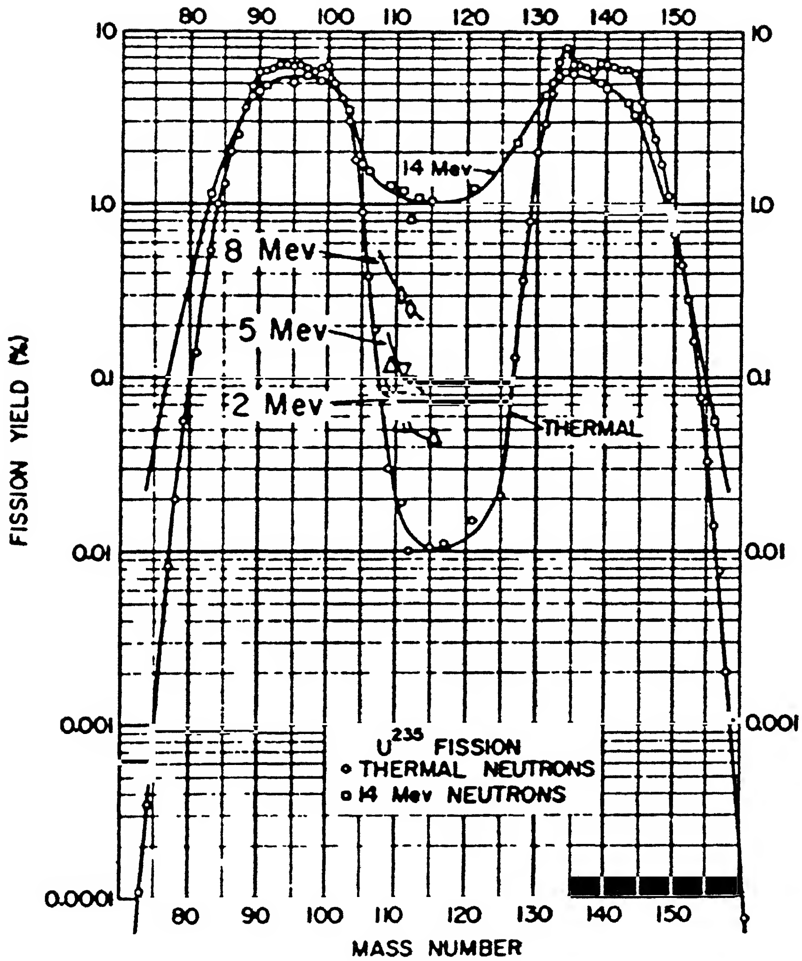


Fig 2.1 : MASS YIELD CURVE FOR FISSION OF U-235 BY THERMAL, 2 MeV (fission spectrum), 5 MeV, 8 MeV AND 14 MeV NEUTRONS.

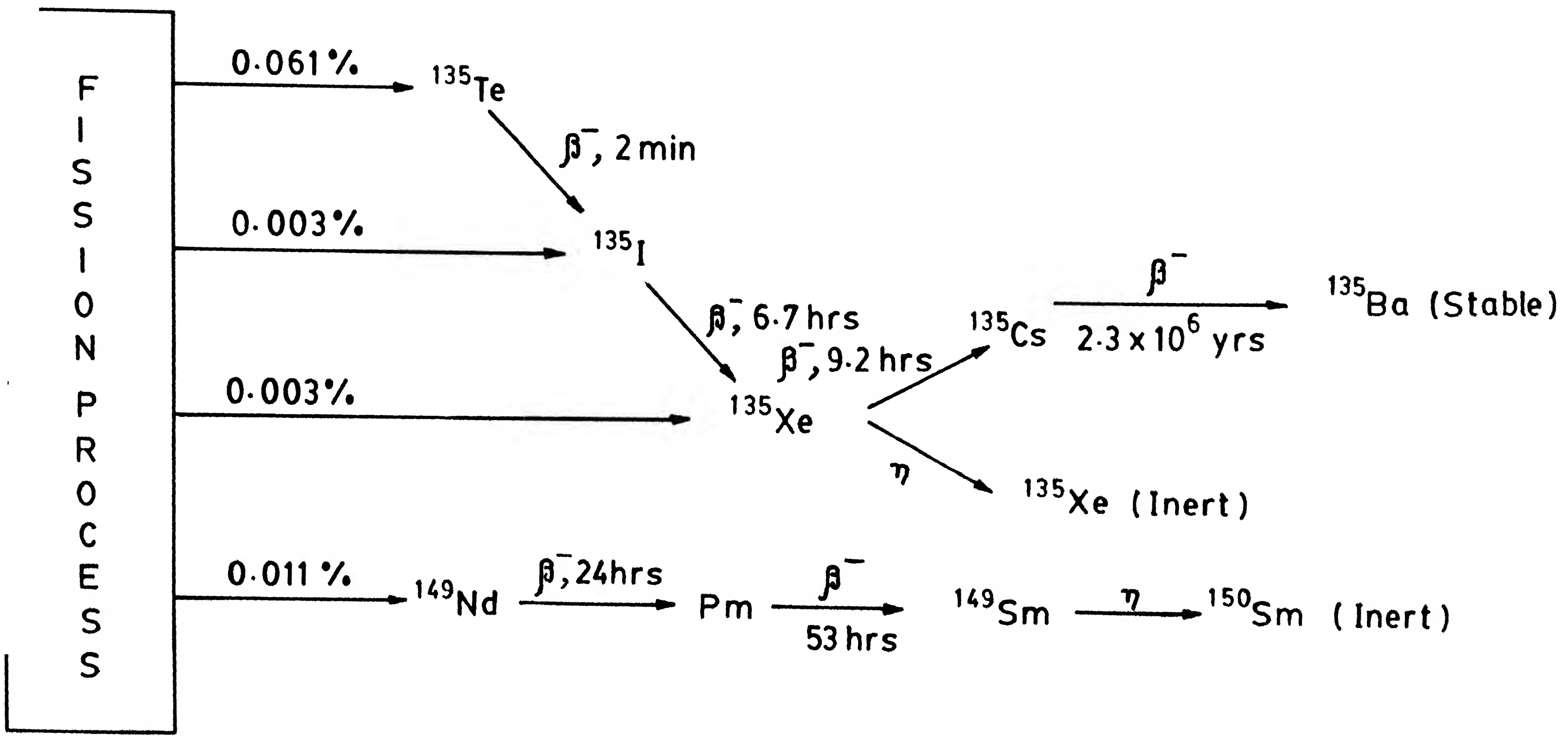


FIG. 2.2 DECAY SCHEMES FOR XENON-135 AND SAMARIUM-149 FISSION PRODUCTS

Table 2.1. THERMAL NEUTRON ABSORPTION CROSS SECTION OF SOME SELECTED REACTOR MATERIALS

MATERIAL	CLASSIFICATION	MICROSCOPIC ABSORPTION CROSS SECTION σ_a, barns
HEAVY WATER	MODERATOR	0.003
ZIRCONIUM	CLADDING	0.18
URANIUM-235	FUEL	678
BORON	CONTROL POISON	759
PLUTONIUM-239	FUEL	1014
SAMARIUM-149	POISON	48000
XENON-135	POISON	2870000

2.3. Poison Kinetics Equations

The mathematical model for Xenon control considerations is based on the behavior of radioactive species depicted in Fig.2.2. Since Te-135 decays so rapidly to I-135 compared to other decay schemes, we can assume that Te-135 is "non-existent" and I-135 is produced directly from fissions with the same yield fraction as Te-135. Then, writing *production* terms with a positive sign and *removal* terms with a negative sign, the time rate change of I-135 and Xe-135 concentrations can be represented by the following equations :

$$\frac{dI(t)}{dt} = \gamma_I \Sigma_f \phi - \lambda_I I \tag{2.1}$$

$$\frac{dX(t)}{dt} = \gamma_X \Sigma_f \phi + \lambda_I I - \lambda_X X - \sigma_X X \phi \tag{2.2}$$

where I(t), X(t) = time dependent Iodine and Xenon concentrations

ϕ = thermal neutron flux, n $cm^{-2}s^{-1}$

λ_I = 2.9×10^{-5} sec^{-1}, the decay constant of Iodine

λ_X = 2.1×10^{-5} sec^{-1}, the decay constant of Xenon

γ_I = 0.056, the yield fraction of Iodine

γ_X = 0.003, the yield fraction of Xenon

Σ_f = macroscopic fission cross section of the fuel

σ_X = $2.87 \times 10^{-18} cm^{-2}$, microscopic absorption cross section of Xenon.

The initial conditions are given by the assumption that the reactor is initially in a steady state , *i.e.*, the time rate

change of the state variables is zero :

$$\frac{dI(0)}{dt} = 0 \tag{2.3a}$$

$$\frac{dX(0)}{dt} = 0 \tag{2.3b}$$

2.4. The Xenon - Iodine Phase Plane (XIPP)

The kinetic behavior of the Xenon poison can be represented on the XIPP shown in Fig 2.3. Curve OE is the locus of the equilibrium point for increasing flux values. When the flux level of the steady state reactor is abruptly brought down to a very low value, production of Xe-135 and I-135 from the fissions stop. However, I-135 and Xe-135 already present in the reactor (from past fissions) continue to β-decay to Xe-135 and and Cs-135 respectively (Fig 2.2). The former decay rate being greater than the latter ($\lambda_I > \lambda_X$), Xe-135 concentration in the reactor begins to build up. Eventually, the reactor gets depleted of enough I-135 to make the production rate of Xe-135 smaller than its removal rate, so that Xe-135 build-up stops at some peak value and begins to decrease, finally tending to zero. This dynamic behavior is graphically depicted on the XIPP by the trajectory ABO. Trajectories such as ABO are called *zero-flux shutdown trajectories.*

Depending on the available Xenon-override capability, reactors have a maximum allowed level of Xenon concentration, Xe_{max}, during or after the shutdown process is complete. The line

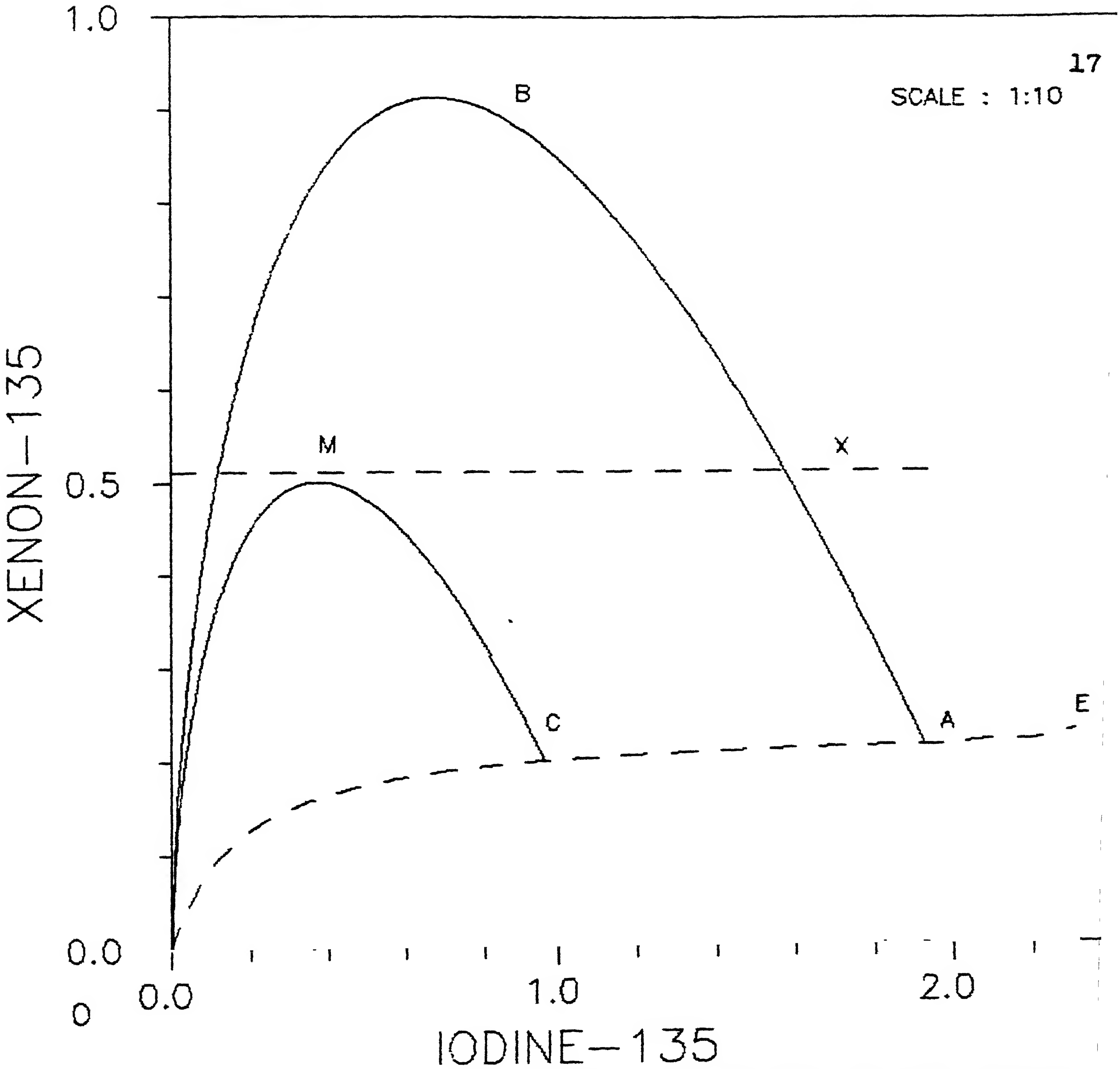


FIG.2.3 : THE XENON-135 IODINE-135 PHASE PLANE (XIPP).

$Xe=Xe_{max}$ is labeled MX on the XIPP. Point C on the equilibrium line OCAE is such that MX is tangent to its zero-flux shutdown trajectory (CMO) at M. Thus, for any equilibrium point lying within the portion OC of the equilibrium line, the reactor can be shutdown abruptly without the danger of Xenon concentration exceeding the prescribed limit (given by line MX). However, for operating points beyond C on the equilibrium line, it is certain that some portion of the corresponding zero-flux shutdown trajectory lies *above* the line MX. During this period of excess poison concentration the reactor cannot be restarted because there is insufficient override capability.

2.5. The Need For An Optimal Control

In large thermal reactors, accumulation of a large quantity of Xenon within a short time following immediate shutdown can overwhelm the reactor's override capability. Depending on the steady state power level, the reactor may not be available for restarting for the next 30-50 hours. This is an unacceptable situation, especially in applications such as nuclear submarines. While it is possible to neutralize the poison with excess fuel loading, the additional expenditure on fuel will severely effect the economy of the reactor operation. Hence we seek some control means to check the Xenon build-up. An elegant solution to this problem is to program the neutron flux during a finite shutdown period so that excess reactivity is continually available. In this regard, we may define optimality in two ways :

(a) A flux program which minimizes the post-shutdown Xenon concentration in the reactor;

(b) A flux program which takes the least amount of time to complete, under the restriction that the post-shutdown Xenon level does not exceed a prescribed maximum limit.

Of the two definitions given above, (b) is a practical criterion for an optimal flux program since, once the Xenon override capability is achieved, further reduction in Xenon level is superfluous and, therefore, time required to complete the flux programming may be minimized instead. In the analysis to follow, this definition of optimality is used.

2.6. Optimal Control Solutions

The time optimal solution problem is solved using the Pontryagin's techniques. The definitions and theorems are rewritten as applicable to the Xenon-Iodine Phase Plane. Time optimal trajectories which conform to the bound imposed on the maximum allowable Xenon level are shown to exist on the XIPP. These trajectories are then generated for all physically realizable steady state operating points. The trajectory corresponding to the point of interest is selected by trial-and-error procedure from the set of trajectories thus generated.

2.6.1. Theorems For Time Optimal Processes On The XIPP

The definitions and theorems developed by Pontryagin *et al* are written here as applicable to the XIPP. Necessary conditions for a minimum time solution are presented.

Definitions

Control Variable : The Control Variable is the flux ϕ, which is restricted to the admissible range $0 \leq \phi \leq \phi_{max}$. Thus, ϕ is in the admissible control range Φ where,

$$\Phi \in [0,\phi_{max}]$$

State Variables : The state variables are the time dependent Xenon and Iodine concentrations in the reactor core, represented on the ordinate and the abscissa, respectively, of the XIPP.

$$x_1(t) = I^{135}/\Sigma_f \tag{2.4}$$

$$x_2(t) = Xe^{135}/\Sigma_f \tag{2.5}$$

As a convenient short hand the following vector notation for the state variables is used :

$$\mathbf{x} = (x_1, x_2).$$

System Equations : These equations represent the time rate change of Iodine and Xenon concentrations (the state variables). From eqs.(2.1) and (2.2), and the state variables defined above, we can write :

$$\dot{x}_1 = \gamma_I\phi - \lambda_I x_1 \tag{2.6}$$

$$\dot{x}_2 = \gamma_X\phi + \lambda_I x_1 - \sigma_X\phi\, x_2 - \lambda_X x_2 \tag{2.7}$$

Using vector notation, these equations may be written as

$\dot{\mathbf{x}} = (f_1, f_2) = \mathbf{f}(\mathbf{x}, \phi)$

Adjoint Variables : The Adjoint Variables are represented by p_1 and p_2.

$\mathbf{p} = (p_1(t), p_2(t))$

Hamiltonian : The Hamiltonian is denoted by H and defined by the following equation :

$H = (\mathbf{f}, \mathbf{p}) = f_1 p_1 + f_2 p_2$

Adjoint Equations : The adjoint equations are defined as

$$\dot{p}_i = -\partial H/\partial x_i \quad ; \quad i = 1,2 \qquad (2.8)$$

Allowable State Space : The central idea of the control procedure is to prevent the Xenon level from exceeding a prescribed limit represented by the $Xe = Xe_{max}$ line (MX) on the XIPP. Thus the allowed state space is defined as any point on the XIPP that lies on or below the line MX. All state space trajectories must be confined to the allowable state space to be acceptable.

Reverse Time Trajectories : Reverse time T is defined as $T = t_1 - t$, where t_1 is a constant and t is the normal time. This definition is used to modify the system equations before the Maximum Principle is applied to determine the time optimal trajectories, which will be in reverse time.

Theorems

Consider some point $x = x_o$ at time $t = t_o$ in the XIPP. Let $x(t)$ for $t_o \le t \le t_1$ be the time optimal trajectory originating at x_o, corresponding to an optimal flux program $\phi(t)$ for $t_o \le t \le t_1$, which transfers the system state from $x = x_o$ at $t = t_o$ to $x = x_1$ at $t = t_1$. Then, the following theorems set the conditions for time optimal solutions.

<u>Maximum Principle</u> : Let $\phi(t)$, $x(t)$ and $p(t)$ be such that they satisfy the equations (2.6), (2.7) and (2.8). Then, the maximum principle requires that, for $\phi(t)$ and $x(t)$ to be time optimal solutions,

(a) the adjoint vector $p(t)$ be continuous and nonzero

(b) the function $H(p,x,\phi)$ for $t_o \le t \le t_1$ attain a supremum at the point $\phi = \phi(t)$; *i.e.*,

$$H = \sup_{\phi \in \Phi} H(p, x, \phi)$$

(c) H is a positive constant (say H_o), *i.e.*,

$$H = H_o \ge 0.$$

<u>Transversality Condition</u> : Let x_o and x_1 be fixed points on the XIPP, where $x_1 \in$ CMO. For $t_o \le t \le t_1$, let $\phi(t)$ and $x(t)$ represent the time optimal flux program and its corresponding trajectory on the state space, and $p(t)$ represent the adjoint vector satisfying the conditions of the Maximum Principle. Let $x_2 = f(x_1)$ be the

equation of the target curve, and $g(x) = dg/dx_1$ represent the tangent to the target curve, for all $\mathbf{x} \in$ CMO. Then, $\mathbf{p}$ is said to satisfy the transversality condition at the curve CMO if $\mathbf{p}(t_1)$ is orthogonal to $g(x_1)$.

The theorem on the transversality condition states that for $\mathbf{x}(t)$ to be the time optimal trajectory, the adjoint vector should satisfy the conditions of the Maximum Principle as well as the transversality condition at $\mathbf{x}(t_1) \in$ CMO.

<u>Jump Condition</u> : A trajectory has to be confined to the allowable state space to be acceptable. Hence, upon encountering the $Xe = Xe_{max}$ line, the trajectory cannot cross it but has to follow one of the two other possible optimal trajectories in reverse time labeled DEJ and DEK in Fig.(2.4). The point $E = \mathbf{x}(\tau)$ is the meeting point of the trajectory and the line MX, and is known as the *junction point*. τ is known as the *junction time*. At any such junction point, the following jump conditions must hold true in order that the trajectory is time optimal.

$$p_1(\tau^-) = p_1(\tau^+) \neq 0$$

$$p_2(\tau^-) = p_2(\tau^+) + \eta$$

$$H(p(\tau^-), \mathbf{x}(\tau^-), \phi(\tau^-)) = H(p(\tau^+), \mathbf{x}(\tau^+), \phi(\tau^+))$$

where η is some real number.

2.6.2. Unrestricted State Space Trajectories

Before proceeding to evolve a method of solution to the optimal shutdown problem, the bound on the allowable Xenon level

is relaxed to find the unrestricted state space trajectories. The reason for this step is explained later. Once the unrestricted trajectories are found, the constraint on Xenon is imposed to yield the restricted time optimal trajectories.

Consider some point $\mathbf{x} = \mathbf{x}_o$ at $t = t_o$ on the XIPP which has to be transferred in a time optimal way to $\mathbf{x} = \mathbf{x}_1$ at $t = t_1$, where $\mathbf{x}_1$ belongs to the target curve. The Maximum Principle allows a number time optimal trajectories to be generated from $\mathbf{x}_o$, since there can be a number of sets of the adjoint variables $p_1(0)$ and $p_2(0)$ which satisfy the condition that the Hamiltonian is a positive quantity. But only one of these trajectories satisfies the transversality condition at the trajectory - target curve meeting point, $\mathbf{x}_1$. Other trajectories are of interest only for different Xenon override capabilities. Thus, finding out the trajectory of interest is a trial-and-error procedure involving determination of the correct set of $p_1(0)$ and $p_2(0)$ values.

A more efficient way of finding the time optimal trajectories is to solve the problem in reverse time defined earlier. The theorems and definitions for reverse time are exactly the same as for normal time. The constant t_1 is chosen such that, the reverse time $T = 0$ coincides with $\mathbf{x}(T) = \mathbf{x}_1$. Clearly, any $\mathbf{x}$ belonging to the target curve is known. The adjoint vector $\mathbf{p}(t_1)$ is determined by the transversality condition and the arbitrary positive value of H that can be assigned. Thus, the boundary conditions for $\mathbf{x}$ and $\mathbf{p}$ are completely specified for all $\mathbf{x} \in$ CMO. The Maximum Principle

can now be applied to the system equations written in reverse time, to generate time optimal trajectories that fill the state space. There is thus a unique trajectory passing through every physically realizable point on the XIPP. From among this set, the trajectory passing through the point of interest is found using a trial-and-error procedure.

In the mathematical treatment of the optimal control problem to follow, the reverse time approach is used. This choice is justified by the following reasoning. It is practical to assume that the Xenon override capability of the reactor varies very slowly with time. On the other hand, the steady state operating power level of the reactor may be assumed to vary frequently within a wide, albeit known, range. Hence, instead of finding a particular trajectory from a given operating point to a fixed target curve, it is more convenient to pick a trajectory of interest from those originating from the target curve. In the remainder of the analysis to follow, the system equations, adjoint equations, and other functions are written in terms of reverse time denoted by the symbol T.

System Equations :

$$\dot{x}_1 = dx_1/dT = \lambda_I x_1 - \gamma_I \phi \tag{2.9}$$

$$\dot{x}_2 = dx_2/dT = \lambda_X x_2 + \sigma_X x_2 \phi - \lambda_I x_1 - \gamma_X \phi \tag{2.10}$$

Equilibrium-line Equation : Equilibrium line is the locus of te steady state operating points for various loadings of the reactor. The Equation to this line on the XIPP can be obtained by equating

$\dot{x}_1$ and $\dot{x}_2$ to zero in the system equations and eliminating ϕ from the two resulting equations :

$$x_2 = \lambda_1(\gamma_1 + \gamma_2)x_1 \; / \; (\gamma_1\lambda_2 + \sigma_x\lambda_1 x_1) \qquad (2.11)$$

Hamiltonian :

$$H = p_1(\lambda_I x_1 - \gamma_I\phi) + p_2(\lambda_X x_2 + \sigma_X x_2\phi - \lambda_I x_1 + \gamma_X\phi) \qquad (2.12)$$

$$\partial H/\partial\phi = -\gamma_I p_1 + p_2(\sigma_X x_2 - \gamma_X) \qquad (2.13)$$

Adjoint Equations :

$$dp_1/dT = \dot{p}_1 = -\partial H/\partial x_1 = -\lambda_I p_1 + \lambda_I p_2 \qquad (2.14)$$

$$dp_2/dT = \dot{p}_2 = -\partial H/\partial x_2 = -p_2(\lambda_X + \sigma_X\phi) \qquad (2.15)$$

Initial Conditions : The initial conditions for the problem in reverse time are determined from the equation of the target curve, the transversality condition at the target curve and the initial value of the flux.

Target Curve Equation : The target curve is described by the following equations :

$$x_2 = f(x_1) = \beta x_1^{\alpha} x_{2M}^{(1-\alpha)} - \frac{x_1}{1-\alpha} \qquad (2.16)$$

$$df(x_1)/dx_1 = \alpha\beta x_1^{(\alpha-1)} x_{2M}^{(1-\alpha)} - \frac{1}{1-\alpha} \qquad (2.17)$$

where $x_{2M} = Xe_{max}$

$\alpha = \lambda_X \div \lambda_I$

$$\beta = \left[\frac{1}{\alpha^{\alpha}} + \frac{\alpha^{\alpha-1}}{1-\alpha} \right]$$

Transversality Condition : The transversality condition to be

satisfied by the trajectory at the target curve can now be put in the form :

$$P_1 + P_2 \frac{df}{dx_1} = 0 \qquad (2.18)$$

Initial Value Of Flux : From equation (2.13) it is clear that $\partial H/\partial\phi$ is not dependant on the value of flux. Therefore, H is a linear function of ϕ. According to the Maximum Principle, for the trajectory to be time optimal, the the Hamiltonian must attain a supremum for all possible values of flux. In order to satisfy this condition, the following relations must hold true :

$$\phi = 0 \text{ for } \frac{\partial H}{\partial \phi} < 0 \qquad (2.19)$$

$$\phi = \phi_{max} \text{ for } \frac{\partial H}{\partial \phi} > 0 \qquad (2.20)$$

Thus, the control variable (flux) is operated at the extremes of its allowable range (0 to ϕ_{max}) depending on the sign of a function, eq.(2.13). Switching from one extreme to the other takes place when $\partial H/\partial\phi$ changes sign. Such an operation is referred to as *bang-bang control*.

In reverse time solution, T = 0 corresponds to the end of the flux program in normal time. The flux has to be reduced essentially to zero at the end of the program in normal time, just after the switching took place. When the sense of time is reversed, this implies that the initial value of the flux in reverse time solution should be ϕ_{max}.

<u>Reverse-Time</u> <u>Solutions</u> : The system and adjoint equations, (2.9-2.10 and 2.14-2.15), are linear first-order differential equations, which are solved below for a constant value of flux.

$$x_1(T) = AC_1 + C_2 \tag{2.21a}$$

$$x_2(T) = BD_1 + AD_2 + D_3 \tag{2.21b}$$

$$p_1(T) = (E_1 / A) - (E_2 / B) \tag{2.21c}$$

$$p_2(T) = p_{20} / B \tag{2.21d}$$

where x_{10}, x_{20}, p_{10}, and p_{20} are the initial values,

$$x_{10} = x_1(T=0)$$

$$x_{20} = x_2(T=0)$$

$$p_{10} = p_1(T=0)$$

$$p_{20} = p_2(T=0)$$

A and B are time dependant quantities given by

$$A = e^{\lambda_1 T}$$

$$B = e^{(\sigma_x \phi + \lambda_x)T}$$

and the other constants have expressions as follows :

$$C_1 = x_{10} - \gamma_I \phi / \lambda_I$$

$$C_2 = \gamma_I \phi / \lambda_I$$

$$D_1 = x_{20} - (\lambda_I x_{10} - \gamma_I \phi)/F - (\gamma_I + \gamma_X)\phi/(F + \lambda_I)$$

$$D_2 = (\lambda_I x_{10} - \gamma_I \phi)/F$$

$$D_3 = (\gamma_I + \gamma_X)\phi/(F + \lambda_I)$$

$$E_1 = p_{10} + \lambda_I p_{20}/F$$

$$E_2 = \lambda_I p_{20}/F$$

$$F = \sigma_X \phi + \lambda_X - \lambda_I$$

Now, the time optimal trajectories can be found for any point lying on the target curve. As already discussed, flux is initially assigned the value ϕ_{max}. A point $\mathbf{x} \in$ CMO is chosen such that eq.(2.16) is satisfied. Using these values of x_1 and x_2, and some arbitrary positive value of H, eqs.(2.12) and (2.18) are solved simultaneously for the adjoint variables p_1 and p_2. A value of H = 0 cannot be chosen because, eqs.(2.12) and (2.18) would then yield a relation between x_1 and x_2 which, in general, would be inconsistent with eq.(2.16). In Sec.2.6.4,the numerical value used in the numerical solution is given. The values of $\mathbf{x}$ and $\mathbf{p}$ thus found are substituted in eq.(2.13) to determine the sign of $\partial H/\partial\phi$. If $\partial H/\partial\phi$ has changed sign, then the flux is switched to zero; else, T is incremented, new values of $\mathbf{x}$ and $\mathbf{p}$ are determined using eqs.(2.21) and the process is repeated.

2.6.3. Allowable State Space For Time Optimal Flux Switching

The reverse time flux program sets down the following two conditions for the first switching to take place :

$$\frac{\partial H}{\partial \phi} = 0 \tag{2.22}$$

$$\frac{d}{dT}\left(\frac{\partial H}{\partial \phi}\right) \leq 0 \tag{2.23}$$

Substituting eq.(2.13) and using eqs.(2.21) in the above two equations, the following two expressions will be obtained :

$$-\gamma_I p_1 + p_2(\sigma_X x_2 - \gamma_X) = 0 \tag{2.24}$$

$$\gamma_I \lambda_I p_2 \left(\frac{p_1}{p_2} - 1 + \frac{\gamma_X \lambda_X}{\gamma_I \lambda_I} - \frac{\sigma_X x_1}{\gamma_1} \right) \tag{2.25}$$

Eliminating p_1 from eqs.(2.24) and (2.25), the following inequality will result :

$$\gamma_I\lambda_I p_2 \left[\frac{\sigma_X x_2}{\gamma_I} - \frac{\gamma_X}{\gamma_I} - 1 + \frac{\gamma_X\lambda_X}{\gamma_I\lambda_I} - \frac{\sigma_X x_1}{\gamma_I}\right] \leq 0 \qquad (2.26)$$

From eqs.(2.21), it is clear that p_2 must have the same sign as p_{20} which is positive. Therefore, eq.(2.26) implies that the region on the XIPP where time optimal flux switching is allowed is given by the relation :

$$x_2 \leq x_1 + \left[\frac{\gamma_I + \gamma_X}{\sigma_X} - \frac{\gamma_X\lambda_X}{\sigma_X\lambda_I}\right] \qquad (2.27)$$

The second term on the right of relation (2.27) (enclosed in brackets) is a constant, say G. The line $x_2 = x_1 + G$ is shown as the line Γ in Fig.(2.5). Time optimal switching above line Γ is thus not allowed. However, since the Maximum Principle allows an arbitrary value of flux along the line Γ ($\partial H/\partial\phi = 0$), any region not accessible through one pulse control can be reached *via* the line Γ, along which flux has the form,

$$\phi_\Gamma = \frac{x_1(2\lambda_I - \lambda_X) - \lambda_X G}{\sigma_X x_1 + 2\gamma_I - \gamma_X\alpha} \qquad (2.28)$$

where α is defined in eq.(2.16) and

$$G = \left[\frac{\gamma_1 + \gamma_2}{\sigma_2} - \frac{\gamma_2\lambda_2}{\sigma_2\lambda_1}\right]$$

2.6.4. Restricted State Space Trajectories

The unrestricted state space trajectories were generated without any constraint on the Xenon level built up during the

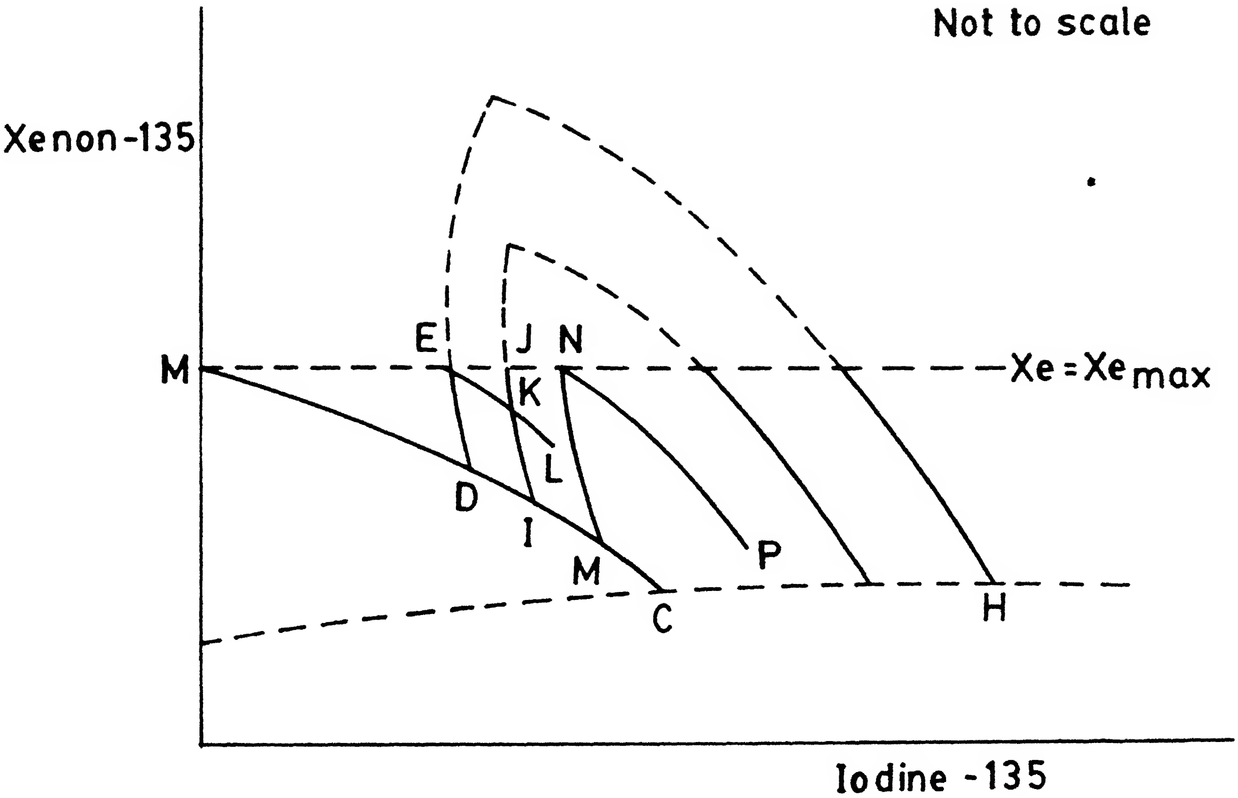


FIG. 2·4 ALTERNATIVES AT THE BOUNDARY LINE IN THE RESTRICTED STATE SPACE

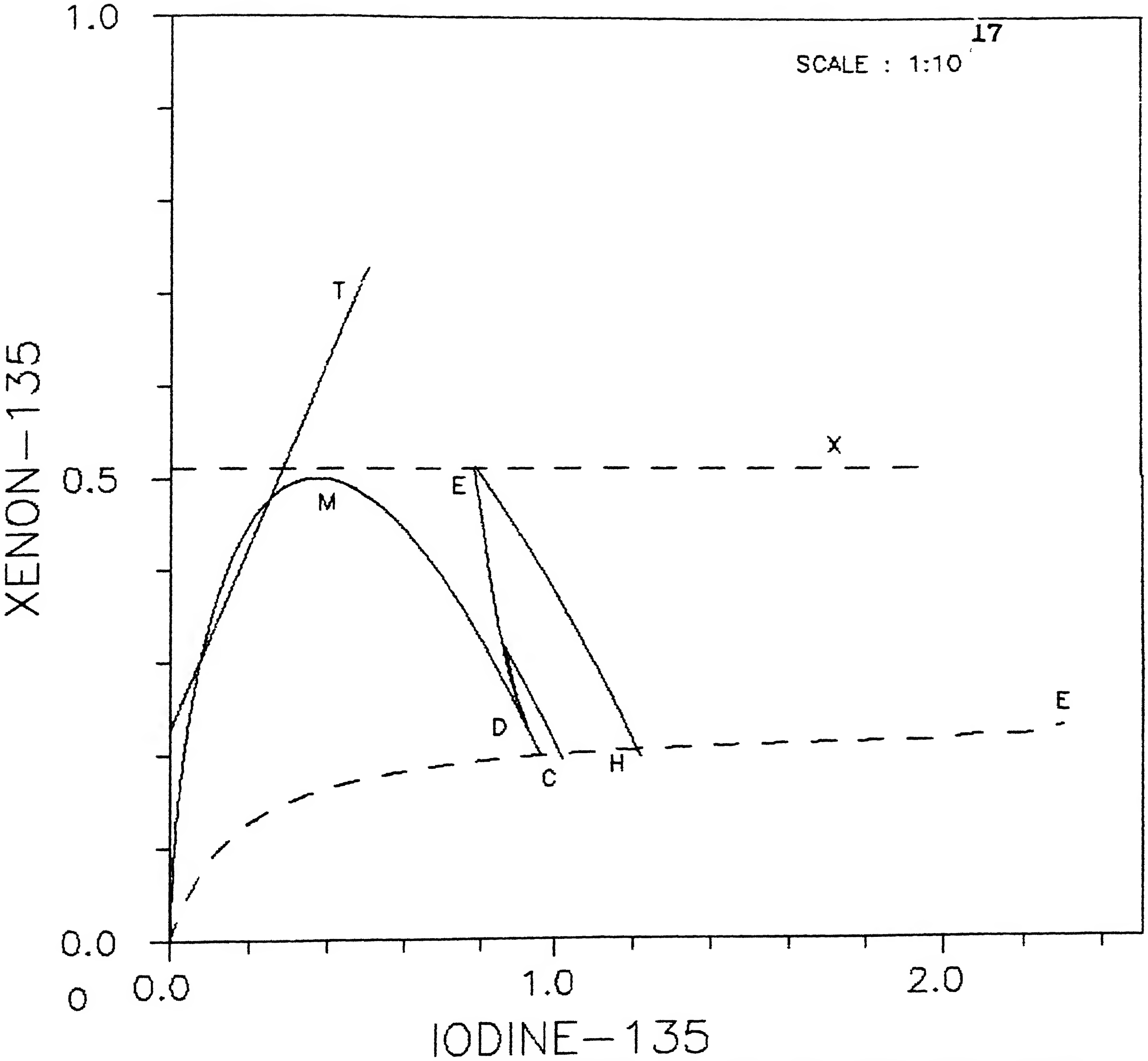


FIG.2.5 : TIME OPTIMAL TRAJECTORIES IN THE RESTRICTED STATE

course of the flux program. Consider for example, trajectory DEFGH (Fig.2.4). The portion EFG which lies above line MX represents a reactor condition with insufficient override capability, and is therefore unacceptable. DE and GH are still time optimal in the sense that they take the least time in transferring the reactor state from D to E and G to H respectively; also, they are acceptable from Xenon considerations as they lie within the allowable region. However, as will be seen soon, time optimal trajectories from points such as H will not employ the portion ED.

Consider the junction point $E(\tau)$. The trajectory cannot proceed along EF since poisoning-out must be avoided. So, there are three other possible paths which the trajectory can now take :

(a) along the boundary line MX (EJ)

(b) along a reflected path such as EKL

(c) stop at E.

The case (c) is unacceptable. In cases (a) and (b), the jump conditions must hold true for time optimality.

The line MX represents a constant value of Xenon concentration. Therefore, along the boundary line,

$$\dot{x}_2(\tau^+) = 0 \tag{2.29}$$

This condition is satisfied if flux jumps discontinuously to some low value. It should be noted that $\partial H/\partial\phi$ is not equal to zero at E since switching did not take place there in the unrestricted state

space. The jump condition of requires that $p_1(T)$ be continuous, but allows a jump in $p_2(T)$. However, since $\dot{x}_2(t^+)$ is zero, the Hamiltonian, given by $H = p_1\dot{x}_1 + p_2\dot{x}_2$ is not affected by the value of p_2. Thus the requirements of discontinuous jump in the value of flux, and continuous H are contradictory. The trajectory therefore cannot take a path along the boundary as in (a).

The possibility (b) is also not acceptable. To demonstrate this, assume that the jump conditions at E are satisfied when the trajectory follows the path DEKL. IKJ is some other time optimal trajectory. Then, in real time, when the trajectory starts from L and reaches the intersection point K, a decision has to be made to follow a time optimal path, since KI is also time optimal and is the best path from K to the target curve. Hence, the trajectory cannot reflect along a path such as EKL.

It is now clear that the operating point cannot be moved from the target curve to the boundary along a path such as DE. However there exists a unique unrestricted time optimal trajectory which has a switching point exactly on the boundary (MNP). For this curve, $\partial H/\partial \phi = 0$ at the junction point so that NP and NG are both time optimal. At G, unlike the situation in (a) above, $\dot{x}_2(\tau^+) \neq 0$ leaving us free to choose a proper value of $p_2(\tau^+)$ to make the hamiltonian a continuous function, thereby, satisfying all the jump conditions. From G, H can be reached in a time optimal way along GH which is a portion of the zero flux shutdown trajectory for point H. Since $\dot{x}_2 = 0$ along the boundary, the flux can be

determined using the equations :

$$\dot{x}_1 = \lambda_I x_1 + \gamma_I \phi \tag{2.30}$$

$$\dot{x}_2 = 0 = \lambda_x x_{2M} + \sigma_x x_{2M}\phi - \lambda_I x_1 - \gamma_I \phi \tag{2.31}$$

where $x_{2M} = Xe_{max}$.

Eqs.(2.30) and (2.31) can be solved for x_1 and ϕ to obtain :

$$x_1 = A\phi + B \tag{2.32}$$

$$\phi = \left[\frac{x_{1N} - B}{A} + B\right] e^{(\lambda_I - \frac{\gamma_I}{A})T} + \frac{B\lambda_I}{A} \tag{2.33}$$

where $A = \dfrac{\sigma_2 x_{2M} - \gamma_2}{\lambda_I}$

$B = \dfrac{\lambda_2 x_{2M}}{\lambda_I}$

$x_{1N} = x_1$ at point N.

2.6.5. Digital Simulation

A computer program for the Xenon shutdown problem formulated in the earlier sections was developed. The steps involved in the computing procedure is depicted in the flowcharts given in Figs.(2.6) and (2.7). The starting point for the program is to assign a suitable value to the maximum allowable Xenon concentration Xe_{max}. This fixes the $Xe=Xe_{max}$ line (MX) and the target curve (CMO) on the XIPP, Fig.(2.3). The target curve and the equilibrium lines are then solved simultaneously to find the intersection point C. A number of starting points lying on the portion CM of the target curve are chosen and, for each point, the reverse-time optimal trajectories are generated using eqs.(2.21).

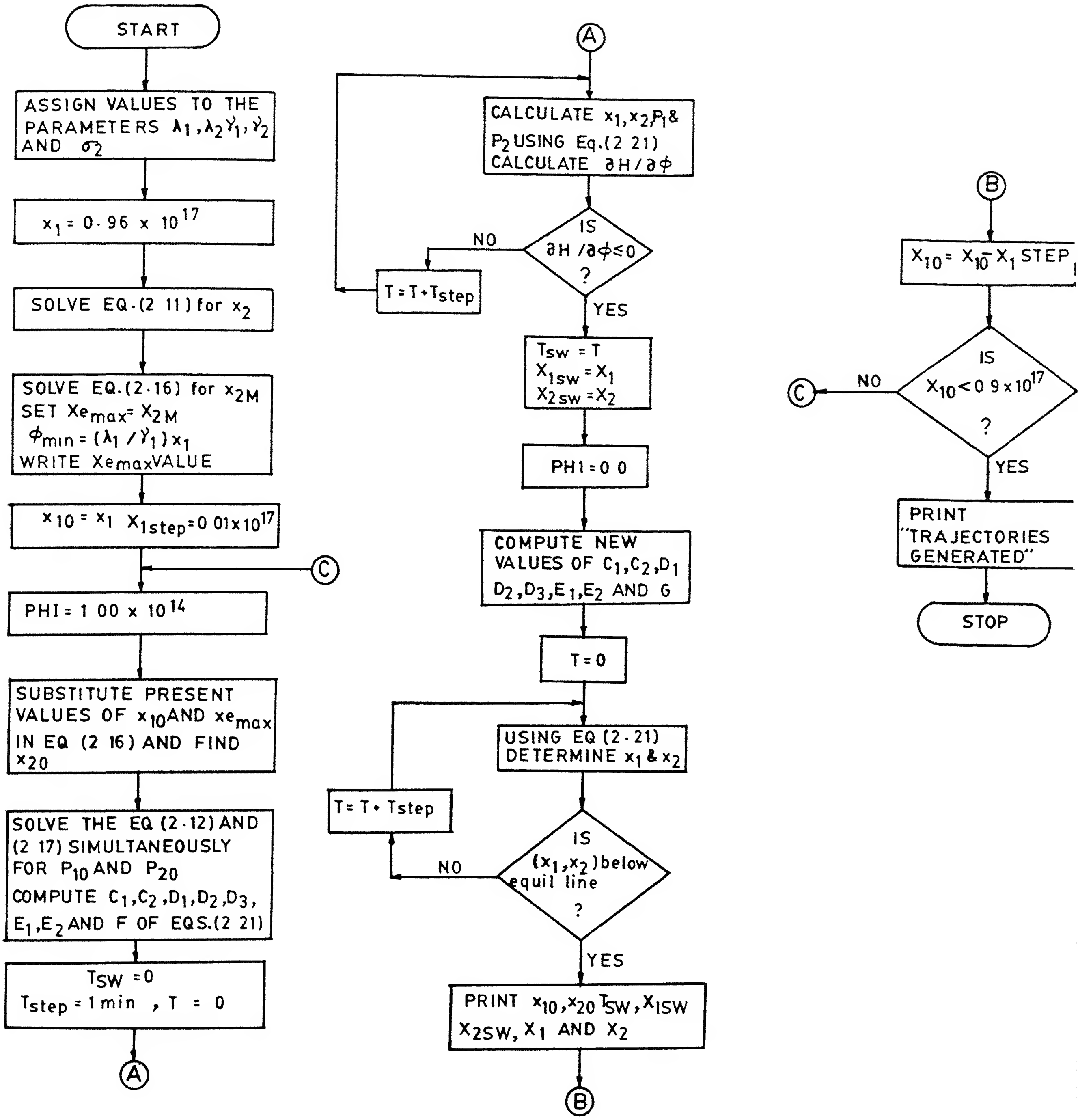


FIG. 2.6 FLOWCHART FOR GENERATING THE REVERSE TIME TRAJECTORIES ON THE XIPP

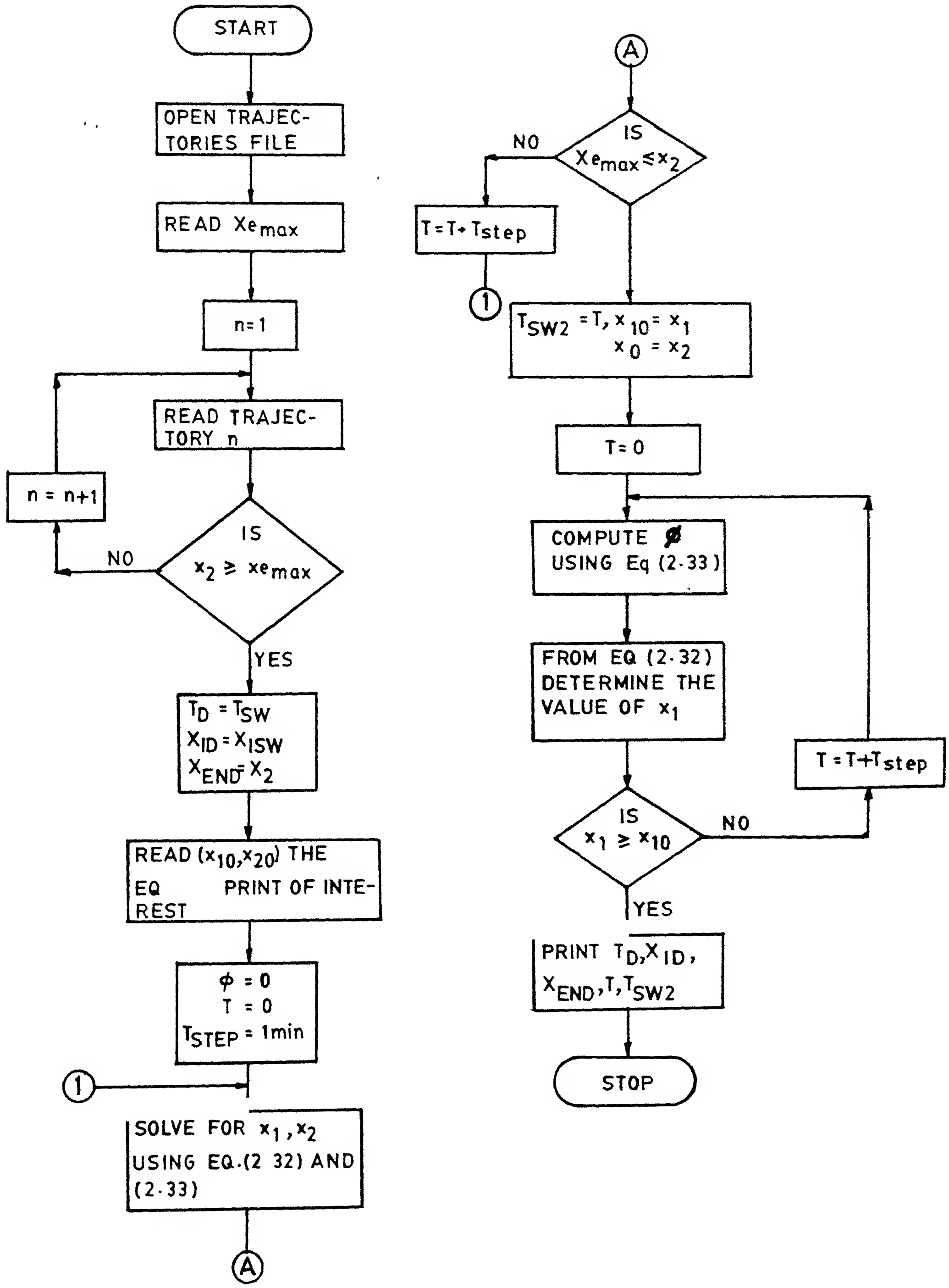


FIG. 2.7 FLOWCHART FOR GENERATING TRAJETORIES FOR POINTS LYING BEYOND ONE PULSE REGION

Initially only the unrestricted time optimal trajectories are generated. Eqs.(2.21) are solved for new values of x_1, x_2, p_1, and p_2 at ten-minute time-step increments. At each step, the sign of $\partial H/\partial\phi$ is checked. When it becomes negative, the flux is set to zero and the new values of the coefficients are calculated. The procedure continues to find x_1 and x_2 values in the subsequent intervals, until the point (x_1, x_2) falls below the equilibrium line. The time-step is reduced to one minute in the interval where $\partial H/\partial\phi$ becomes negative and (x_1, x_2) falls below the equilibrium line in order to obtain a better estimate of the switch point and the intersection point.

Once all the restricted state space trajectories are generated, the next step is to locate the trajectory with switch point lying exactly on the line MX, which is labeled DEH in Fig.2.5. DEH is taken to be simply the trajectory with x_2 at switch point lying closest to Xe_{max}. Thus the points lying on the portion CH of the equilibrium line are accessible through one-pulse control. However for points beyond H, a certain portion of the trajectory lies along the line MX, and the portion DE is common to all of them. The complete trajectory is found by choosing a steady-state operating point lying beyond point H, setting the flux to zero and tracing the zero flux shutdown trajectory until it cuts the line MX. Along the line MX, the flux varies according to equations (2.30) and (2.31) till the operating point is brought to E. The flux is then set to ϕ_{max} and operation

continues along ED until the target curve is reached. The flux is then permanently set to zero.

2.7. Test Results And Conclusions

The Xenon Shutdown problem was formulated and solved using the Maximum Principle. The following data for a point reactor model were used to generate the test results :

$Xe_{max} = 5.0 \times 10^{16}$ cm^{-2}

$\gamma_I = 0.061$

$\gamma_X = 0.002$

$\lambda_I = 2.8525 \times 10^{-5}$ sec^{-1}

$\lambda_X = 2.1088 \times 10^{-5}$ sec^{-1}

$\sigma_X = 2.70 \times 10^{-18}$ cm^2

$\phi_{max} = 1.00 \times 10^{14}$ n cm^{-2} sec^{-1}

$H = 1.0 \times 10^{12}$

The computer code was written in FORTRAN 77 and run on hp9000,850s computer systems. The execution time was negligible and increased by small amounts as the number of trajectories generated increased.

Results of a test run are presented in Fig.2.8. Trajectories were generated for two steady state operating points taken as examples. From the results it is observed that :

(a) The solution is optimal in a sense that the flux program takes the minimum time to complete and the Xenon concentration in

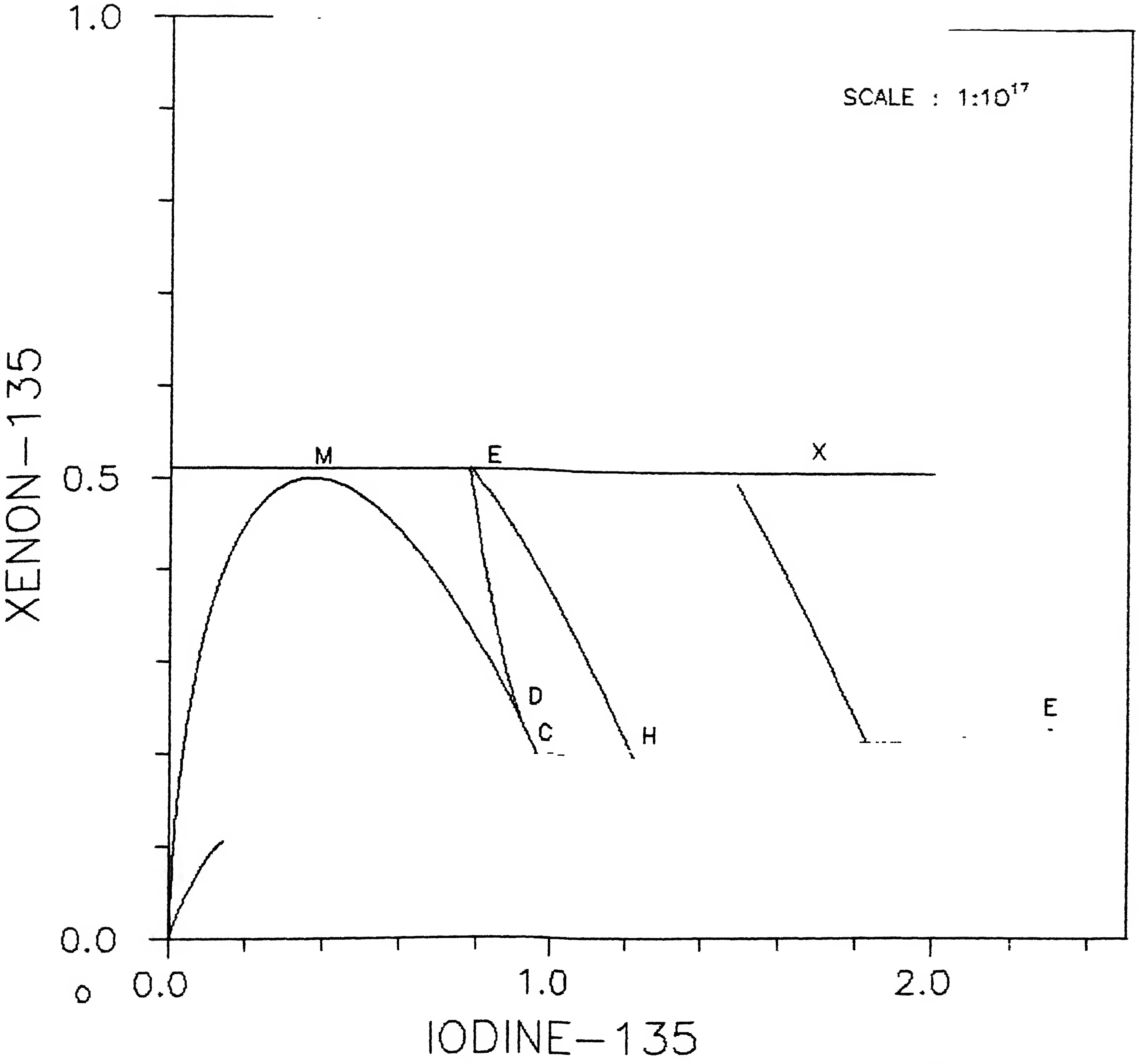


FIG.2.8 : EXAMPLES OF TWO TIME OPTIMAL TRAJECTORIES OBTAINED FROM THE DIGITAL SIMULATION STUDIES.

the reactor at any time is restricted to the specified maximum allowable value.

(b) Depending on the specified value of Xe_{max}, no flux programming is required for steady state flux values below a certain maximum. For example, for $Xe_{max} = 5.0 \times 10^{6}$, no flux programming is required if the steady state flux value is not greater than 0.96×10^{14} n $cm^{-2}s^{-1}$.

(c) Single pulse control is required for steady state operating points which give zero flux shutdown trajectories lying above the target curve. The normal range of the steady state operating points lies in the one-pulse control region. For the numerical example chosen, this range extends from 0.96×10^{14} to 0.56×10^{17} n $cm^{-2}s^{-1}$.

(d) For high flux values ($> 5.6 \times 10^{16}$ n $cm^{-2}s^{-1}$), in order to reach the target curve without violating the constraint on the permissible Xenon level, operation along the $Xe=Xe_{max}$ line was found to be necessary. Flux along this line is variable (decreasing) and will never exceed the value of ϕ_{max}.

(e) Unrestricted state space trajectories that necessitate operation along the line Γ would ultimately reach steady state operating points that correspond to very high steady state flux values ($> 10^{18}$ n $cm^{-2}s^{-1}$). Such points are practically impossible to attain. Hence these trajectories need not be considered.

(f) A change in the value of the hamiltonian, H, does not effect the results in any manner, as long as it is a positive constant.

(g) Since bang-bang type control is not feasible in actual reactor operation, a separate program was written to determine the effect of varying the flux in (i) step wise manner and (ii) exponential manner. In the latter case, the flux was assumed to vary in uniform steps taking a total of ten minutes to switch from one extreme to the other. In the latter case, the flux was assumed to rise with a time-constant of 80s and decay with a time-constant of of 2s. In both these cases, there was negligible change in the values of the state variables and compliance with the Xenon constraint is still maintained.

The above results form the basis for the knowledge base of the expert system is discussed in the next chapter.

CHAPTER 3

AN EXPERT SYSTEM FOR SHUTDOWN CONTROL OF NUCLEAR REACTORS

3.1. Introduction

Analytical solutions to the optimal shutdown control problem were obtained in Chapter-2. A typical optimal shutdown flux program for high flux steady state operating points was found to consist of sequential operation at zero power, variable (decreasing) power, and finally full power. Since these solutions were generated using a simplified reactor model, they are not directly applicable for the optimal control purposes of an actual reactor system. For example, the single-pulse bang-bang type control obtained for medium-flux steady state operating points cannot be directly carried out in practice since there is a constraint on the maximum rate of change of reactor flux level. If this constraint is violated, the reactor may undergo dangerous power excursions. It is possible to rectify such drawbacks by applying certain operational and engineering knowledeges about the reactor system to the time-optimal shutdown programs obtained in the previous chapter so that a heuristically near optimal solution will be obtained. Such a system, based on artificial intelligence (AI) techniques, is developed in this chapter. A brief introduction to AI and AI based systems is given. The expert system, which belongs to a class of intelligent systems is defined and the architecture of a general expert system is discussed. Expert systems in the Nuclear Industry and their implementation issues are also briefly discussed. An expert system for the

shutdown control of nuclear rectors is developed.

3.2. Artificial Intelligence (AI)

Artificial Intelligence is a relatively new field in computer science concerned with understanding the nature of intelligent action and constructing computer systems capable of such actions. It embodies the twin objectives of furthering basic understanding and making computers more sophisticated in the service of humanity. The most widely accepted definition of Artificial Intelligence is that due to Marvin Minsky : "Artificial Intelligence is doing with computers that which if done by humans is said to require intelligence".

The Phenomena comprising an intelligent action involves many activities such as problem solving, perception (visual, auditory, tactile), learning, symbolic activity, creativity, language etc. The starting point for artificial intelligence is the capability of the computer to manipulate symbolic expressions that can represent all manner of things (or, structures). To work on a task, a computer must have an internal representation in its memory, for example, symbolic description of a room for a moving robot. The description also includes all the basic programs for testing and measuring the structure, plus all the programs for transforming the structure into another one in ways appropriate to the task.

Given the representation of the task, the next step is to adopt some problem solving method so that there is a finite chance

of accomplishing the task. For example, in pursuit of a solution, a sequence of candidates may be generated each being tested for solutionhood, or, sequences of operations may be tried out to construct a path from initial condition to the desired one.

To carry out the tasks of representation and problem solving, an operating frame is required, *i.e.*, a machine that provides data structures, operations on these data structures, a programming language, and an interpreter for carrying out programs. An operating frame is often called the executive, or control, structure and may be viewed as a total architecture. Determining an appropriate architecture for a general intelligent action is same as determining an appropriate programming language for artificial intelligence systems, which calls for ability for processing symbolic information, such as words and phrases. Aware of this need John McCarthy invented LISP (for list processing) in 1959. The basic commands of LISP can sort out symbols, can build up lists, can determine the truth or falsity of a function, and can match the IF-side of a production rule. In general, LISP's goal is to evaluate something and return a value. In 1972, Alain Cormerauer at the University of Marseilles began development of PROLOG (for programming in logic), based on a different principle. While a LISP program consists of a series of commands that manipulate symbols, a PROLOG program consists of statements of facts and rules. This information is used by asking questions about it.

The basic paradigm of an intelligent action is that of a

search through a space (called a problem space) for a goal situation. The search is combinatorial, each step offering several possibilities leading to a cascading of possibilities in a branching tree. What keeps the search under under control is the knowledge, which tells how to choose, or narrow, options at each step. A general intelligent system will have immense amounts of knowledge. Discovering the relevant knowledge as the solution attempt, or search, progresses is not a trivial problem. Unlike the combinatorial explosion characteristic of problem search space, this one involves a fixed, though large, data base, whose structure can be carefully tailored by the architecture to make search efficient. This encoding and access constitutes the final ingredient of an intelligent system.

Historical Background : The term *artificial intelligence* was coined by John McCarthy in 1958; this year is often quoted as the birth of Artificial Intelligence. The framework was established through the 1960s and early 1970s. As the field of Artificial Intelligence developed, a whole new range of specialized areas evolved including natural language processing, natural vision, image recognition, automatic learning, robotics and expert systems.

3.3. Expert Systems (ES)

Expert systems are a subset of intelligent systems. An Expert System may be defined as a computerized process or program~~s~~ that attempt~~s~~ to emulate the human thought process associated with the application of substantial knowledge of specific areas of

expertise to solve finite, well-defined problems. These computer programs contain human expertise (called heuristic knowledge) obtained either directly from human experts or indirectly from published literature, as well as general and specialized knowledge that pertains to a specific situation. Expert systems have the ability to reason using formal logic, to seek information from a variety of sources including data bases and the users, and to interact with conventional programs to carry out a variety of tasks including sophisticated computation.

3.3.1. Expert System Architecture

An expert system has three principal components : the inference engine, the knowledge base, and the man-machine interface. These components are described briefly below. The relationship of an expert system to the environment in which it functions is shown in Fig.3.1.

Inference Engine (IE) : The inference engine of an expert system is in charge of manipulating data and arriving at a conclusion. The required data or information is gathered from the knowledge base component of expert system. Other sources of knowledge base include associated data bases and the user. The IE uses rules of logic to draw inferences or conclusions for the processes involved; the knowledge input to the IE forms the basis for the conclusions reached. The search process for a solution proceeds in a controlled manner in accordance with the programmed strategy. Two most widely used search techniques are forward chaining (forward reasoning) and backward chaining (backward reasoning). In

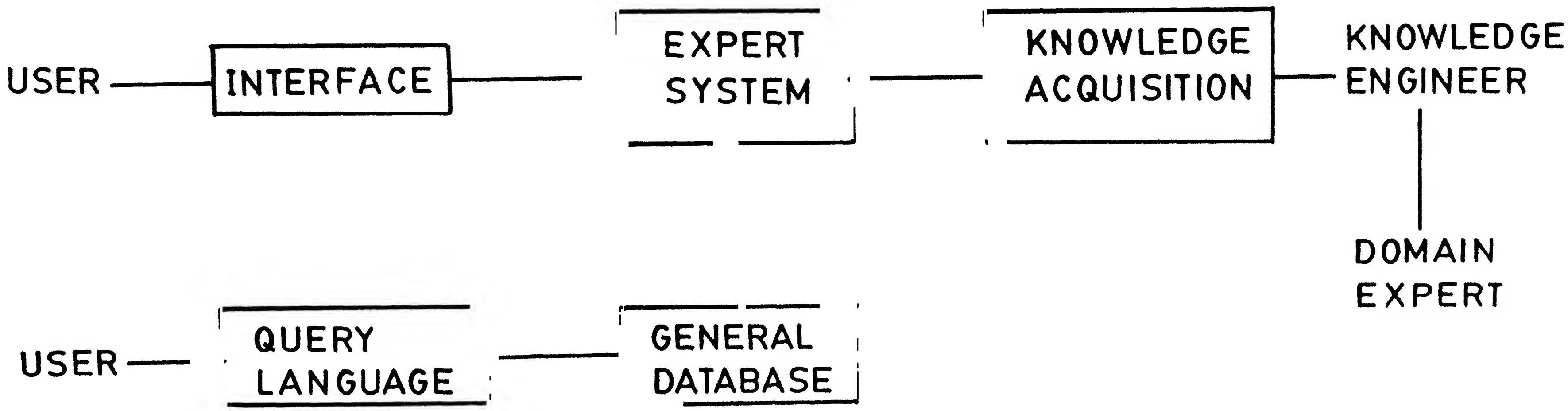


FIG. 3.1 THE RELATIONSHIP OF AN EXPERT SYSTEM TO THE ENVIRONMENT IN WHICH IT FUNCTIONS

forward chaining the system from forward chaining the system reasons forward from a set of known facts and tries to infer the conclusions or goals. In backward chaining the system works backward from conclusions or goals and attempts to find supporting evidence to verify their conclusions. In many cases, backward chaining systems are more efficient than true forward chaining systems because they tend to reduce the search space and arrive at a conclusion quickly. However, many advanced expert systems use use both forward and backward chaining.

<u>Knowledge Base (KB)</u> : The knowledge base acquires knowledge through knowledge acquisition software tools from a knowledge engineer who, in turn, acquires the knowledge from a domain expert, books, publications and other sources. Here, domain refers to a specific field of knowledge. The knowledge is then encoded in rules, frames or other computer representations of knowledge. This information describes a methodology for solving the problem as a human expert would solve it. Because an expert system is only as good as its knowledge, proper collection and representation of knowledge is critical for successful implementation and operation of an expert system. Once the expert system has been built, the relationship between the knowledge base and the knowledge engineer shown in Fig 3.1 can be ended. However, since the domain normally continues to develop, the relationship normally continues. At present, most expert systems depend upon a specialized knowledge base. They will become more usable with existing data processing systems when they become able to draw on general databases (Fig [illegible] in the immediate computer system or on a network.

Query languages are adding intelligence to the task of searching a database. An intelligent front end capable of guided search could home in on the required information quickly.

Man-Machine Interface : The interface between the human and the expert system must translate the user input into the computer language, and it must present the conclusions and explanations to the user in written or graphical form. The user usually works through a keyboard interface in a formal language. As natural - language capability becomes available, this interchange becomes easier. Eventually speech recognition and generation may replace the keyboard. The interface should also include an editor to assist in adding to or changing the knowledge base.

Other Important Features : Some expert systems have what is called an *Explanation Facility* which has been recognized as one of the most valuable features of expert systems. These expert systems contain a certain degree of self-awareness or self-knowledge that allows them to reason about their own operation and to display inference chains and traces of rationale behind their results. The user can take advantage of explanation facilities to request a complete trace for consultation, request an explanation on how a particular goal or subgoal was inferred, or request an explanation why a particular piece information was needed. Explanation generating facilities are also of great use in debugging expert systems and may play a key role in verification and validation of expert systems.

An *expert system shell* is a computer program used to develop an expert system. Early shells were expert systems from which domain-specific knowledge bases had been removed and the mechanism for creating a new knowledge base of user's choice has been made "user friendly". Often a shell also has provisions for changing the reasoning processes of the inference engine to adapt to the specific problem.

3.3.2. Advantages And Disadvantages Of The Use Of Expert Systems

Certain Characteristics of expert systems are unique and generally advantageous :

1. Expertise is available for consultation at remote locations where pertinent human experts are not present
2. They are capable of giving consistent and expert advice in a routine manner and they are not subject to shortcomings of human beings such as fatigue, carelessness *etc*.
3. The inference engine consults the knowledge base and outputs only the relevant information to the user. Consequently, 'information overload' during stressful conditions is avoided. On the other hand, the (optional) use of explanation facility provides the user with the complete basis for the conclusion reached by the expert system.
4. The expert knowledge once acquired for the knowledge base of an expert system is permanent and easy to

transfer whereas, human expertise is lost through retirement and not easy to transfer.

A few shortcomings of expert systems are stated below :

1. They usually deal with static situations. In case conditions change, the knowledge base must be updated accordingly.
2. Expert systems are only as good as the knowledge incorporated into its knowledge base. They are also not able to learn from experience as humans do.
3. Expert systems are unable to solve problems outside their domain of expertise and in many cases are unable to detect limitations of their domain. If an expert system operates outside its domain, it is possible that it may generate incorrect results by utilizing nonapplicable, irrelevant knowledge while searching for a solution.

3.3.3. Fuzzy Expert Systems

Reflecting human expertise, much of the information in the knowledge base of a typical expert system is imprecise, incomplete or not totally reliable. Hence a *certainty factor* is usually given to qualify the conclusion given by the expert system. These factors are an indication of the degree of confidence that the system has in its conclusion. For example, a certainty factor of 0.99 might indicate that the system is 'almost certain' in its conclusion, 0.70 might mean 'likely', and 0.01, 'very unlikely'. Many expert systems in use today compute the certainty factors

using what are essentially probability based methods.

3.4. Expert Systems In The Nuclear Industry

The application of AI technologies, particularly expert systems, to nuclear power plants offer significant advantages. AI technology can greatly expand the capabilities of designers in building safer and economical plants. The use of expert systems in control room activities of a nuclear power plant has the potential to reduce operator error and improve plant operation, reliability and safety. In addition there are numerous non-operating activities such as testing, maintenance, outage planning, equipment diagnostics and fuel management in which expert systems can increase the efficiency and effectiveness of plant and corporate operations.

Applications

The application of expert systems in some major areas of nuclear power plants and their function in these areas are briefly discussed below.

Operator Advisors : Expert systems can provide expert advice to the plant operators through rapid access to a large information base, thereby reducing the potential for operator error. Applications include monitoring normal plant maneuvers and emergency procedures, perform on-line analysis of plant status and provide guidance, data logging and its interpretation, and ensuring the ability to remove residual heat.

Diagnostics : Expert systems can monitor the behaviour of the plant to detect incipient failures and long term gradual deterioration. The plant equipment can be diagnosed for malfunctions or abnormal behaviour.
Example : GenAid is an on-line generator-diagnostic system, designed to diagnose hundreds of conditions with damage potential to the electrical generator and to recommend corrective action for each condition.

Safety : The safety of operation of a nuclear power plant is a top-priority issue due to the magnitude of accident consequences. Expert systems can aid in enhancing the safety of reactor operation by determining compliance with technical specifications and limiting conditions of operation. They can also classify emergency conditions, resolve ambiguous situations and analyse core and containment conditions which will greatly help the safety team to assess the safety of the reactor system. In accident situations, expert systems find applications in real time evacuation, prediction of plume travel and minimization of radiation exposure. Several safety assessment and safety review advisors are currently under development [13].

Outage Planning : In refuelling activities such as fuel reshuffling or optimal fuel handling, the knowledge of human experts is very essential. Their expertise can be incorporated into into the knowledge base of an expert system to perform the refueling activities at a reduced time scale.

Nuclear Training : The fundamental and synergistic relationship between training and expert systems offers an unique oppurtunity to improve training of nuclear power plant personnel. One of the feature that makes expert systems so compatible with diagnostics in nuclear power plants is its ability to explain its reasoning and conclusions for postulated or real conditions given to it.

3.5. Application Of An AI Method To Optimal Shutdown Control

The optimal control laws or control programs obtained for a simplified model may not be suitable for application to actual complex reactor processes. It is necessary that the gap between the theoretical study of the simplified model and its practical application to control of actual reactor system be filled by using various knowledges about the system. For this purpose, AI techniques, particularly AI based expert systems, may provide a practical method for constructing a heuristically near-optimal or practically optimal control system by flexible use of various engineering and operational knowledges about the system. An expert system thus developed would be useful for open-loop consultation purposes. However, for closed-loop process control, automatic decision making and real-time execution are additional requirements of the system.

The design of the proposed knowledge based optimal control method takes into consideration some primary requirements. For every problem situation posed to the computer program, only a single control output must always be generated. If the knowledge base of the expert system is not organized well, it may happen

that no control output is generated as the result of program execution. Hence, a well organized knowledge base is a primary requirement of the expert system. The data incorporated into the knowledge base must be consistent so that inconsistent or contradictory control outputs will not be generated. In case of real time applications, the size of the knowledge base should be small so that the computer program can home in on the required knowldege quickly. It is also desirable that the structure of the knowledge base be simple and easily understandable so that modifications can be made without difficulty. This can be of considerable advantage since the knowledge base and the inference engine can be built as separate modules so that the control logic may be modified flexibly only by modifying the knowledge base and not the computer program.

Basic Concept : Assume that the dynamics of the system to be controlled can be expressed as follows using the state space approach :

$$\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u}) \; ; \qquad \mathbf{x} \in Xc \; , \quad \mathbf{u} \in Uc \tag{3.1}$$

where $\mathbf{x}$ and $\mathbf{u}$ are state and control vectors respectively and Xc and Uc are the associated constraints.

Consider the situation where $f(\mathbf{x}, \mathbf{u})$ and the constraints are completely known and the state vector can be precisely estimated. In addition, suppose that for some optimization problem of the system, we can find an exact optimal control law expressed as,

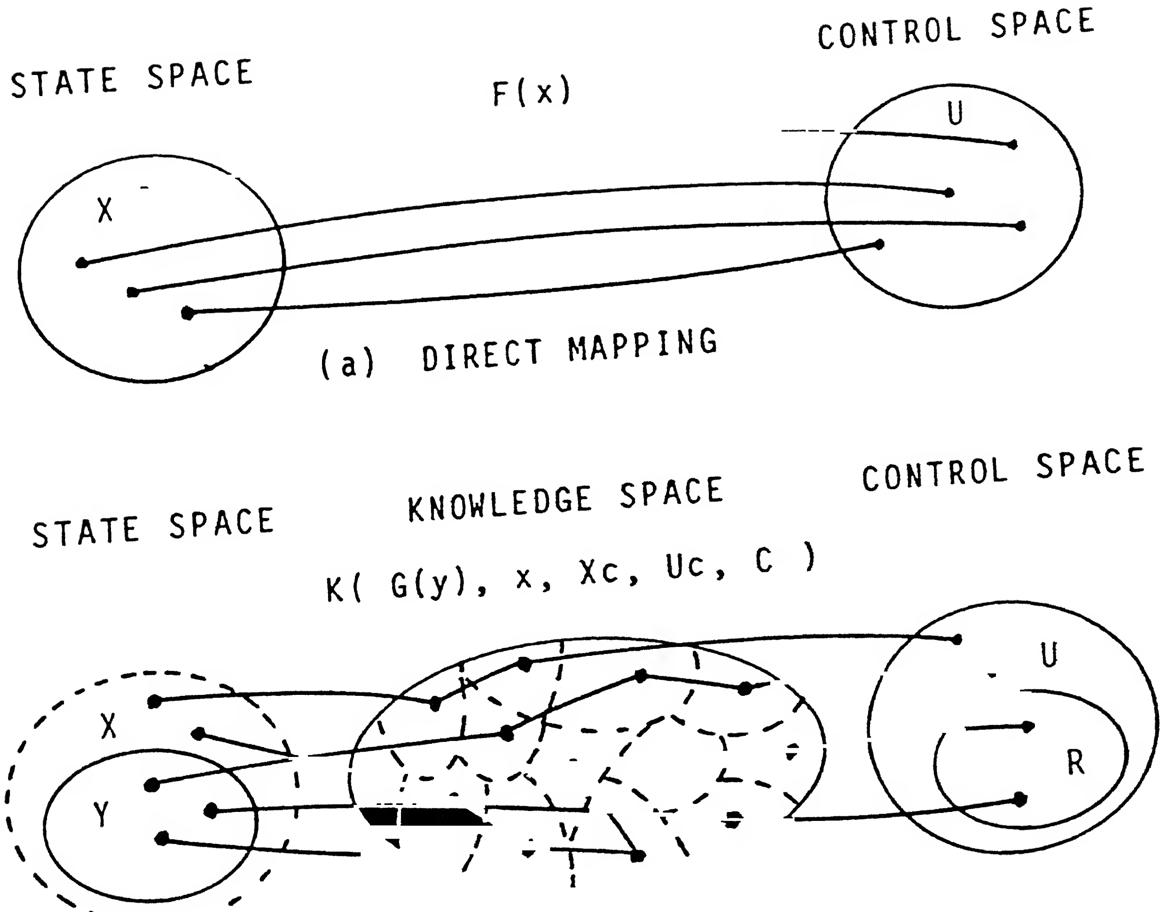
$$\mathbf{u} = F(\mathbf{x}) \; ; \qquad \mathbf{x} \in Xc \; , \quad \mathbf{u} \in Uc \tag{3.2}$$

where F is a direct mapping from state space X onto the control space U. Equation (3.2) is conceptually illustrated in Fig.3.2(a). It is not always possible to find an exact control law such as $\mathbf{u} = F(\mathbf{x})$. The major limitation is set by the simplicity needed for theoretical treatment of the model developed for the physical system. Suppose that, through consideration of the physical phenomena involved or through mathematical judgement, it is possible to build a simplified model of reduced dimension which retains the essential characteristics of the original system, and that it is possible to find an optimal control law for the simplified model thus developed. If $\mathbf{y}$ and $\mathbf{v}$ represent the state and control vectors, respectively, of reduced dimensions, the optimal control law can be written as some mapping G from $\mathbf{y}$ onto $\mathbf{v}$ as follows :

$$\mathbf{v} = G(\mathbf{y}) \; ; \qquad \mathbf{y} \in Yc \; , \; \mathbf{v} \in Vc \tag{3.3}$$

where Yc and Vc are constraints associated with $\mathbf{y}$ and $\mathbf{v}$ respectively.

Since it has been assumed that the essential features of this optimal control for the simplified model are maintained also for the original system, there is a possibility that a knowledge base can be constructed in which near-optimal control rules are derived from the knowledge of the essence of the optimal control law for the simplified model and other pertinent knowledges about the original system. The optimal solutions may be obtained by appropriate combination of these knowledges, symbolically



(b) INDIRECT MAPPING THROUGH INFERENCE

Fig 3.2 Conceptual illustration of the knowledge-based control
(a) Optimal control law (assumed to be unknown)
(b) Knowledge-based near-optimal control using rules

expressed as,

$$\mathbf{u} = H(G(\mathbf{y}), \mathbf{x}, Xc, Xc, C) \qquad (3.4)$$

Here H signifies a combination of knowledges indicated in parenthesis and C signifies supplementary knowledge about the given system. Equation (3.4) is illustrated conceptually in Fig.3.2(b). The dotted lines in the knowledge space indicate fuzziness due to inaccurate estimate of the system state and incomplete knowledge of the control law for the given system.

<u>System</u> <u>Organization</u> : A rule-based model of IF-THEN type is used for knowledge representation. Among various other types of knowledge representation, this has the simplest and most intuitive form of expressing cause-consequence relationships and , therefore, most suitable for control applications. The rules are clssified into four groups to further simplify the description of the rule-base. The four groups are basic control, process dynamics, constraint and adjustment rules. Each of these is described below.

The basic control rules include qualitative and quantitative control rules. A qualitative control rule is an expression like "if X lies in Xr, then rise the reactor power ". A quantitative rule is a statement like "if X lies in Xm, then switch the flux to its maximum value". An appropriate combination of basic rules generate a candidate control vector. The dynamics rule is numerical model of the dynamics of the given system and is used to predict the value of the state and control vector vectors using

the current candidate control vector. The constraints imposed on the state and control vectors are expressed by the constraint rules. In case the predicted state and/or control vector violate the constraints imposed on them, the adjustment rules are used to make necessary modifications of the value of the candidate control vector. The basic control and the constraint rules could be integrated so that a candidate control vector satisfying the constraints is generated directly without the use of adjustment rules. But such a knowledge base would be more complex than in the case where separate rules are used.

At each control time-step, the system state is assumed to be estimated from directly measurable process variables using a state vector estimator. Using the current estimated state vector, a candidate control vector is searched by chaining the basic control rules. Prediction of the state and control vectors are then made using dynamics rule and the candidate control vector. The xandidate control vector and the predicted state and control vectors are checked for their validity. If they are valid, the candidate control vector is executed, otherwise it is modified so that they become valid by repeating the above process of prediction and thereafter using the adjustment rules. The knowledge manager, which includes a simple inference engine for the basic control rules, controls the above mentioned process of generating a valid control vector at each control cycle.

3.7. Expert System Model

The conceptual model of the expert system is shown in

Fig.(3.3). The major components of the expert system, the humans interacting with it, and the interface between man and the machine are indicated with the directed arrows representing direction of information flow. The function of each block and its relation to the other blocks are described below.

<u>Knowledge Engineering</u> : This block represents the key component in the evolution process of the expert system. The simulation program, the optimal control law expressed on the XIPP, and the domain expert are three important sources of knowledge to the knowledge engineer. The function of the knowledge engineer is to select the relevant knowledge from all the sources available to him and to incorporate it into the knowledge base of the expert system using appropriate knowledge representation techniques. Other functions of the knowledge engineer include writing the inferencing strategy into the inference engine, representing the numerical model in the solution analyser, building an explanation facility into the system, debugging and validating the system through dialog with the domain experts, and maintaining the expert system.

<u>Users</u> : The users are the domain experts (who verify the system performance), and the control room operators (who consult the validated and verified expert system during the actual course of reactor operation). Other users include operator trainees who present hypothetical situations to the expert system and learn the nature of the system being controlled from the system responses. The link between the users and knowledge engineering

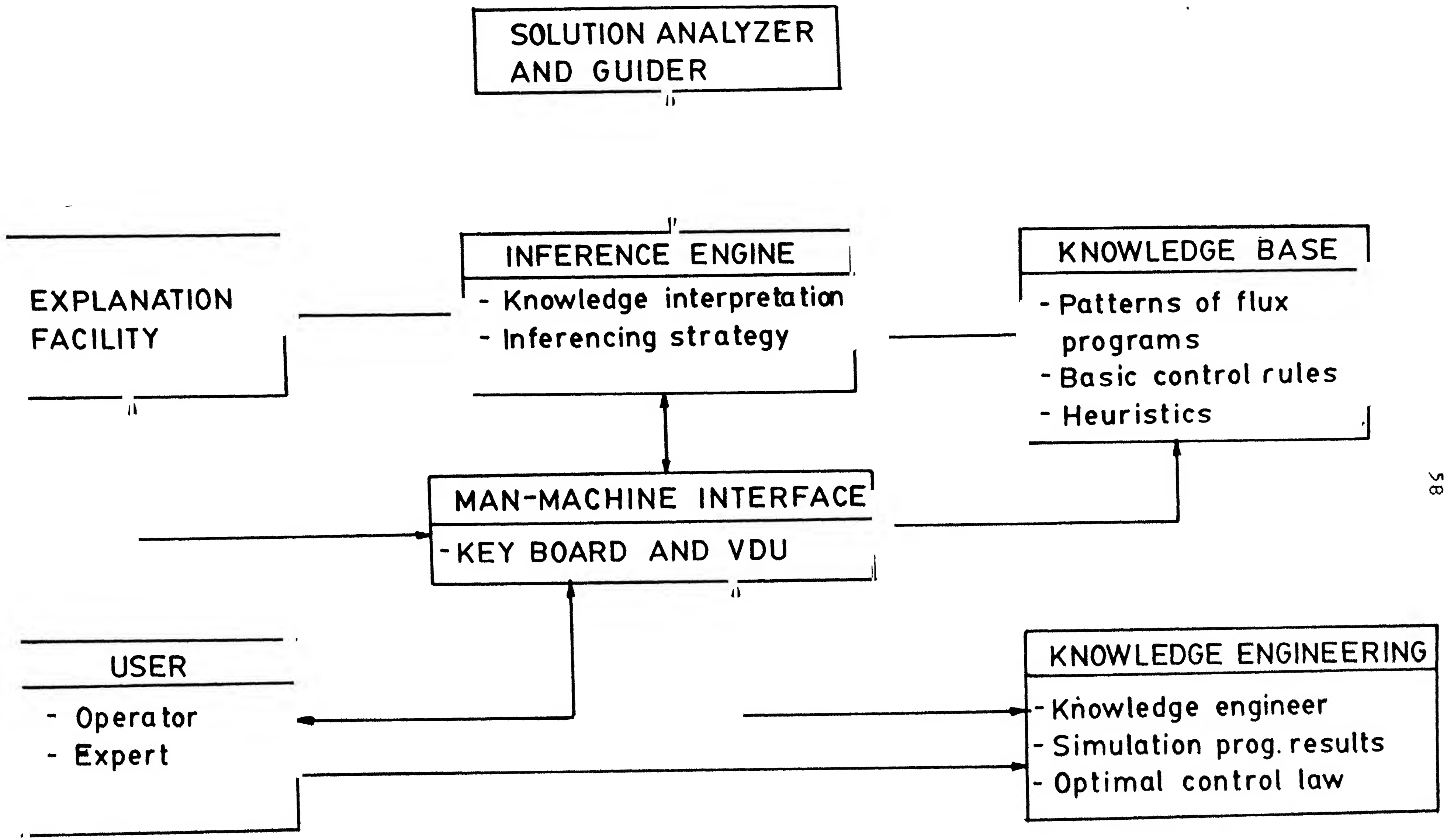


FIG. 3.3 CONCEPTUAL MODEL OF THE EXPERT SYSTEM USED FOR THE SHUTDOWN CONTROL AND SPATIAL OSCILLATIONS PROBLEMS

blocks represents the dialog between the domain expert and the knowledge engineer, which is required to validate the system.

Man-Machine Interface : The two-way communication link between the user and the expert system is provided by the man-machune interface. It represents the keyboard and the visual diaplay unit which facilitate command and information exchange between the user and the expert system. The interface also provides access to the general computing environment so that the editing, printing, memory storage and other facilities may be availed.

Knowledge base : The efficacy of the expert system is critically dependant on the content and structure of the knowledge base. Hence the knowledge engineer must choose the appropriate knowledge representation techniques and tools to build this module.

The first step is to classify the given operating point according to the type of control action required in order to transport it to the target curve in a time-optimal way. Typical patterns of time-programs obtained from the simulation program will aid in this classification. For the example considered in Chapter-2, it was found that if the steady state operating flux was below 0.96×10^{14} n $cm^{-2}s^{-1}$, no flux programming was required; if it was between 0.96×10^{14} n $cm^{-2}s^{-1}$ and 0.56×10^{17} n $cm^{-2}s^{-1}$, one-pulse control was required; and for values above 0.56×10^{17} n $cm^{-2}s^{-1}$, varible flux operation in addition to a single pulse control was required. The steady state operating points belonging to these three regions may be labeled as *low*, *medium*, and *high*

respectively. These three regions are indicated in Fig.(3.4). In the figure, CMO is the target curve and DEF is the reverse-time, time-optimal trajectory with its switch point lying exactly on the $Xe=Xe_{max}$ line (MX). The equations of the zero flux shutdown trajectories for points C and F will determine the region to which the operation point under consideration belongs.

If the operating point lies in region-1, a control action recommending the flux to be set to zero is generated. If it lies in region-3, the pulse width for operation along ED is given along with the values of the state variables at the switch point, E, and the flux is temporarily set to zero. Once the solution analyser (discussed later) detects that the operating point has reached the line MX, an appropriate variable flux operation to reach point D is initiated. For operating points lying in region-2, a suitable interpolated one-pulse control from knowledge of the typical patterns of one-pulse control is recommended as the required control action. These rules which generate the candidate control vector for a given operating point are termed as basic control rules. They are of IF-THEN type where, if the cause matches the IF-part, the THEN-part is triggered as the associated consequence.

Fuzziness may arise in classifying the steady state operating points as low, medium, or high as the total core operation time increases. This is because, after a long opertion time, Xe_{max} for the reactor tends to decrease. Thus some steady state operating points on the borders of these regions may cross over from low to medium and from medium to high. However, since the variation of

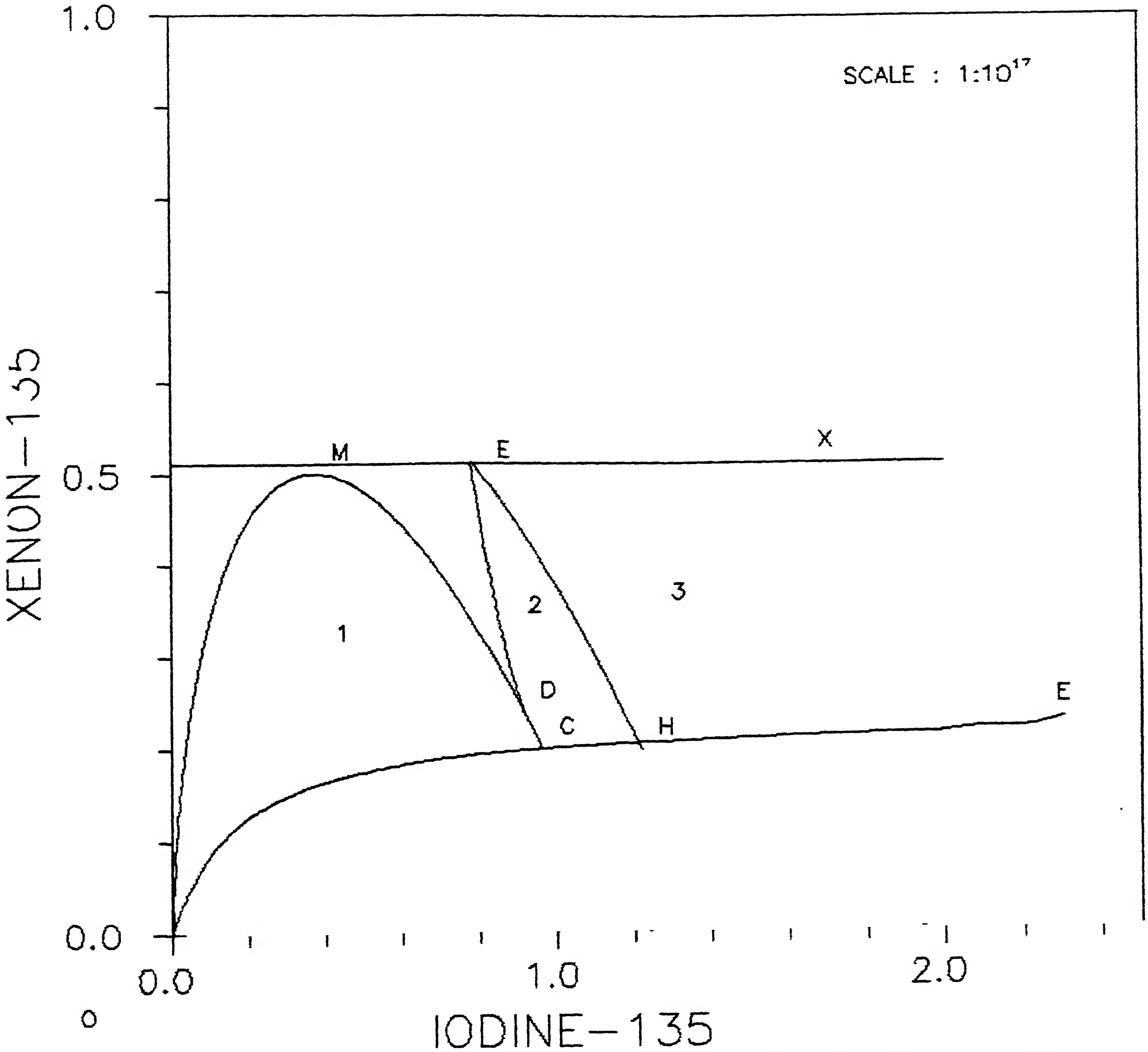


FIG.3.4. : THE ALLOWABLE STATE SPACE DIVIDED INTO THREE REGIONS ACCORDING TO THE CONTROL REQUIREMENTS

```
(defun fp (x1)
  (setq lambda_i 2.8525e-05)
  (setq lambda_x 2.1088e-05)
  (setq gamma_i 0.061)
  (setq gamma_x 0.002)
  (setq sigma_x 2.7d-18)
  (setq point_c 0.963)
  (setq point_h 1.223)
  (setq flux '((The flux is) (low. ) (medium. ) (high. )))
  (setq control '((No) (1-pulse) (> 1-pulse) (control is required. )))
  (setq out nil)
  (if (< x1 point_c)
                    (setq out (append (first flux) (second flux) (first control) (last control))))
  (if (and (> x1 point_c) (<= x1 point_h))
              (block
                    (setq out (append (first flux) (third flux) (second control) (last control) (terpri)))
                    (setq x1 (* 1.0d+17 x1))
                    (setq x2 (* lambda_i (+ gamma_i gamma_x) x1))
                    (setq x2 (/ x2 (+ (* lambda_x gamma_i) (* sigma_x lambda_i x1))))
                    (setq x1 (/ x1 1.0d+17))
                    (setq x2 (/ x2 1.0d+17))
                    (format t "STEADY STATE : x1 = ~s and x2 = ~s" x1 x2)
                    (format t "~%Determining the pulse-width ...~%")
                    (setq zero_flux '(
                                        (3.117) (9.292) (16.283) (24.308) (33.600) (44.467)
                                        (57.417) (73.142) (92.675) (117.867) (152.058) (202.625)))
                    (setq pulse_width '(
                                         (1.467) (4.075) (6.858) (9.858) (13.108) (16.650)
                                         (20.558) (24.925) (29.875) (35.633) (42.575) (51.467)))
                    (setq x2_1 .960)
                    (setq x2_2 .961)
                    (setq x2_3 .962)
                    (setq x2_4 .965)
                    (setq x2_5 .969)
                    (setq x2_6 .975)
                    (setq x2_7 .983)
                    (setq x2_8 .994)
                    (setq x2_9 1.011)
                    (setq x2_10 1.034)
                    (setq x2_11 1.071)
                    (setq x2_12 1.133)
                    (format t "Evaluating IF - THEN forms ... ")
                    (if ((= x2 x2_1)
                          (setq out (append out '(zero-flux) (nth 0 zero_flux) '(pulse-width) (nth 0 pulse_width))))
                    (if (and (> x2 x2_1) ((= x2 x2_2))
                          (setq out (append out '(zero-flux) (nth 1 zero_flux) '(pulse-width) (nth 1 pulse_width))))
                    (if (and (> x2 x2_2) ((= x2 x2_3))
                          (setq out (append out '(zero-flux) (third zero_flux) '(pulse-width) (third pulse_width))))
```

Fig. 3.5 : EXAMPLES OF RULE REPRESENTATION IN THE KNOWLEDGE BASE OF THE EXPERT SYSTEM

Xe_{max} is a gradual process, the effect may be neglected.

Solution Analyser And Guider : The solution analyser incorporates the reactor dynamics model and the constraints imposed on the maximum rate of change of neutron flux (the control variable), the maximum allowed value of Xenon at any time and the maximum allowed value of the control variable, ϕ_{max}. Using the current candidate control vector generated by chaining the basic control rules, the solution analyser uses the numerical model of the reactor system to predict the state and control vector at the next time-step. The current state vector is assumed to be generted directly from measurable process varibles. If the predicted state and control vectors confrom to the constraints imposed on them, the candidate control vector is executed. Otherwise, the candidate control vector is modified in an appropriate manner by the solution guider and a new candidate control vector is generated. The above process of predicting and, if required modifying it again, is repeated.

For example, consider an operating point belonging to region-3. If it has been predicted that the constraint on Xenon will be violated at the next time-step, the flux is raised to an appropriate level such that at the subsequent time-step, the operation point moves along line MX towards E, when the flux is varied according to eq.(2.33). Once point E is reached, flux is set to ϕ_{max} so that operation continues along ED.

Inference Engine : The inference engine consists of a simple knowledge manager which chains the appropriate basic control rules

carried out but it is worth exploring the possibility of developing such a system for PWR designs ranging from very low power test reactors to very high power commercial reactors.

CHAPTER 4

OPTIMAL CONTROL OF SPATIAL OSCILLATIONS

4.1. Introduction

In the previous chapters, the optimal shutdown control of thermal nuclear power reactors to prevent poisoning-out was considered. The control strategy developed consisted of programming the reactor neutron flux to vary in a manner which ensures reactor start-up capability during and after the shutdown process. Such a variation of gross reactor neutron flux to acheive the control objectives is the direct result of using the point, (or, lumped kinetics) reactor model. The model is an adequate representation for the analysis of reactor shutdown problem since, following shutdown, Xenon concentration builds up or decays throughout the reactor core volume in a uniform manner, eliminating the need for including spatial dependance in the poison kinetic equations.

The phenomenon of spatial as opposed to gross Xenon oscillations is another class of Xenon poisoning problems encountered in thermal nuclear power reactors. When such oscillations occur, the total power level of the reactor remains unchanged but, a cyclic variation in power level may build up in various local regions of the core. This is a potentially more damaging situation than poisoning-out as the reactor structural materials may be subjected to excessive temperatures and/or thermal fatigue.

The origin of Xenon induced oscillations, their effects on normnal reactor operation and the need for optimal control methods are discussed in this chapter. The lambda mode solutions to the optimal control problem are obtained and a conceptual model for an expert system based solution to the problem is proposed.

4.2. The Phenomenon Of Xenon Induced Spatial Power Oscillations

The Xenon induced spatial power oscillations are most likely to occur in large, high-power, non-boiling thermal nuclear reactors. Consider such a reactor in a steady state, operating at a high flux level. The Xenon concentration throughout the core will be at a constant level, its production rate being exactly balanced by its removal rate. Suppose that due to some local perturbation, there is a local increase in neutron flux density. The kinetic balance between production and removal of Xenon will be disturbed due to the perturbation, leading to a local increase in Xenon burn-out, thereby rising the local net neutron level which further aids removal of existing Xenon. Thus the local Xenon level gets progressively depleted, at the same time rising the local neutron level (i.e., power level). However, the process does not continue indefinitely since an increase in neutron level increases fission rate, increasing the production rate of Iodine-135 which eventually decays to Xenon, thereby compensating the initial decrease in Xenon level that followed the perturbation. As the total power is being kept constant, an increase in local power level is accompanied by a decrease in power level at some other location in the core. Thus there is a shift in Xenon concentration from one region of the core to the other which is termed as *Xenon tilt*. The Xenon tilt is not stable

because the process catches up with itself through an exactly opposite chain of events thus swinging the tilt in the the opposite direction. The whole sequence of events enumerated above repeat with a time period in the range of tens of hours.

The nature of these oscillations can be seen from Fig. 4.1., as a higher "mode" superimposed upon the fundamental flux shape. The oscillations produce the effect of a moving "hotspot" to the reactor operator. It is possible that the oscillations will diverge producing locally damaging temperature excursions even though the gross power is kept constant. Safety of operation may also be jeopardised in such situations. Hence it is necessary to provide local control rods to check the build-up of local instabilities. An optimal control program is defined as that flux program which is most effective in removing the oscillations and takes the least time to do it.

4.3. Problem Formulation

The aim of the present analysis is to iumplement the control of spatial oscillations in the axial direction. For the sake of simplicity, an infinite slab reactor with an extrapolated thickness L is used. The dynamics are described by the one group diffusion equation with Xenon-Iodine feedback and power feedback as follows :

$$\frac{1}{v}\dot{\phi} = \vec{\nabla}\cdot D\vec{\nabla}\phi - \Sigma_a\phi - X\sigma_x\phi + \nu\Sigma_f\phi + \alpha\Sigma_f\phi(\phi - \phi_o) \qquad (4.1)$$

$$\dot{X} = \lambda_I I + \gamma_x\Sigma_f\phi - \lambda_x X - X\sigma_x\phi \qquad (4.2)$$

$$\dot{I} = \lambda_I\Sigma_f\phi - \lambda_I I \qquad (4.3)$$

where $\vec{\nabla}$ = the laplacian,

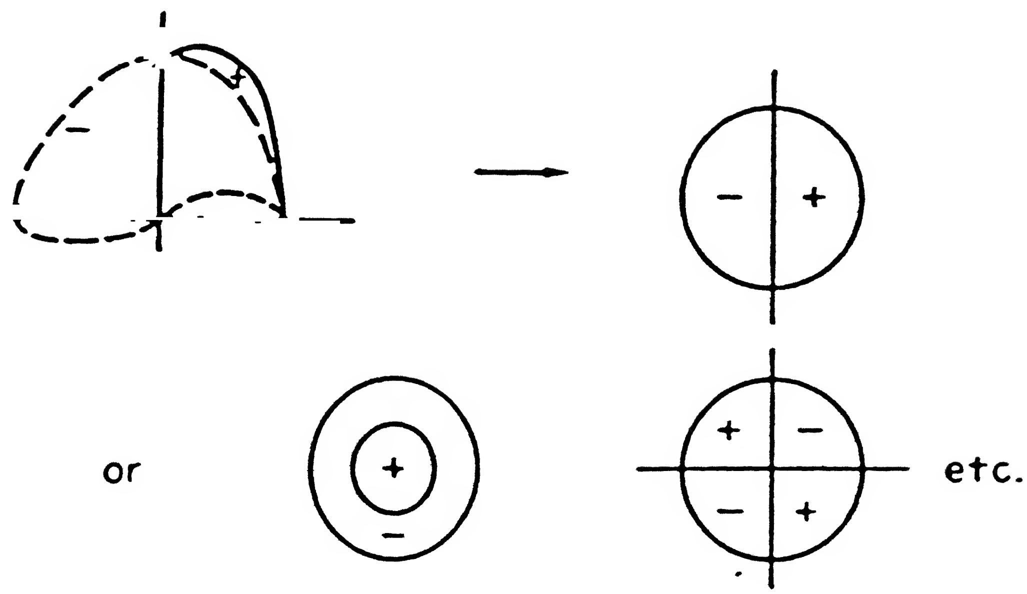


Fig 4.1 : SPATIAL OSCILLATIONS SEEN AS A HIGHER MODE SUPERIMPOSED ON THE FUNDAMENTAL FLUX SHAPE.

D = diffusion coefficient, cm

α = power feedback coefficient,

ν = neutrons emitted per fission,

and other symbols were defined in Chapter 2.

In writing equations (4.1) to (4.3), it has been assumed that the reactor temperature is proportional to the power. For the time scales involved in spatial oscillation considerations, this is a valid assumption. Hence the the temperature feedback of moderator temperature and Doppler effect is represented as the power feedback in the last term on the right hand side in equation (4.1).

In order to simplify the construction of control laws, it is necessary to derive linearized systems around some steady state solutions. To this end, we begin by expanding the flux, Xenon and Iodine densities in small perturbations about their equilibrium values :

$$\phi(r,t) = \phi_o(r) + \delta\phi(r,t) \tag{4.4}$$

$$X(r,t) = X_o(r) + \delta X(r,t) \tag{4.5}$$

$$I(r,t) = I_o(r) + \delta I(r,t) \tag{4.6}$$

Putting equations (4.4) to (4.6) into equations (4.2) and (4.3), and subtracting the steady state balance equations yields,

$$\delta\dot{X} = \lambda_I(\delta I) + \gamma_X\Sigma_f(\delta\phi) - \lambda_X(\delta X) - \sigma_o\phi_o(\delta X) - \sigma_X\phi_o(\delta X) \tag{4.7}$$

$$\delta\dot{I} = \gamma_I\Sigma_f(\delta\phi) - \lambda_I(\delta I) \tag{4.8}$$

In the above two equations, all second order non-linear terms have been neglected. Since it has been assumed that the

perturbations given in equations (4.4) to (4.6) are small, second order non-linear terms are indeed negligible. However, if the oscillation amplitude is large, the non-linear terms may become significant. This situation does not arise if the oscillations are controlled before becoming excessive.

4.4. The Lambda Mode Solutions

The spatial flux, Xenon and Iodine deviations are now expanded in terms of lambda modes [17,18]. The lambda modes are denoted by ψ_i, and are defined by the equation,

$$(- \vec{\nabla}\cdot D\vec{\nabla} + \Sigma_a + \sigma_X X_o)\ \psi_i = \frac{1}{\lambda_i}\ (\nu\Sigma_f)\psi_i \tag{4.9}$$

or,

$$L\psi_i = \frac{1}{\lambda_i} M\psi_i \tag{4.10}$$

where L and M are the destruction and production operators respectively, and λ_i is the criticality of the i^{th} mode, (λ_i has to be distinguished from λ_I, which is the decay constant of Iodine-135). The adjoint equation is defined by

$$L^*\psi_j^* = \frac{1}{\lambda_j} M^*\psi_j^* \tag{4.11}$$

which is related to equation (4.10) by the orthogonality property,

$$\langle\ \psi_j^*\ \frac{M}{\lambda_j}\ \psi_i\ \rangle = \delta_{ij} \tag{4.12}$$

where < > denotes integration over reactor volume and δ_{ij} is the kronecker delta. Since the present analysis uses infinite sl;ab reactor of extrapolated thickness L, equation (4.12) takes the form

$$\int_o^L \psi_j^*\ \frac{M}{\lambda_j}\ \psi_i\ dx = \delta_{ij} \tag{4.13}$$

where M and the modes are functions of x. Assuming $\delta\phi$, δX and δI can adequately be represented by just the fundamental and the first harmonic overtones, the following expansions may be written down,

$$\delta\phi(r,t) = a_0(t)\psi_0(r) + a_1(t)\psi_1(r) \tag{4.14}$$

$$\delta X(r,t) = b_0(t)\Sigma_f\psi_0(r) + b_1(t)\Sigma_f\psi_1(r) \tag{4.15}$$

$$\delta I(r,t) = c_0(t)\Sigma_f\psi_0(r) + c_1(t)\Sigma_f\psi_1(r) \tag{4.16}$$

In equations (4.13) to (4.15), $a_i(t)$, $b_i(t)$ and $c_i(t)$, $i=0,1$ are time-dependent coefficients for the mode amplitudes which must be determined.

By substituting equations (4.13) to (4.15) into equations (4.7) and (4.8), weighting by the adjoint flux ψ_1^* and integrate over the reactor core volume, the following equations expressed in matrix form will be obtained :

$$\begin{bmatrix} \dot{b}_1 \\ \dot{c}_1 \end{bmatrix} = \begin{bmatrix} -\lambda_X - \frac{\nu}{\lambda_1} \langle\psi_1^*\sigma_X\Sigma_f\phi_0\psi_1\rangle \lambda_I b_1 & \lambda_I \\ 0 & -\lambda_I \end{bmatrix} \begin{bmatrix} b_1 \\ c_1 \end{bmatrix} + \begin{bmatrix} \gamma_X - \frac{\nu}{\lambda}\langle\psi_1^*\sigma_X X_0\psi_1\rangle \\ \gamma_I \end{bmatrix} \tag{4.17}$$

By repeating the same process for the flux equation, we obtain the following equation for the first overtone flux,

$$\begin{aligned} \frac{1}{v}\langle\psi_1^*\dot{a}_0\psi_0\rangle + \frac{1}{v}\langle\psi_1^*\dot{a}_1\psi_1\rangle = & - a_1(\Delta\lambda_1)\langle\psi_1^*\nu\Sigma_f\psi_1\rangle \\ & - b_1\langle\psi_1^*\sigma_X\Sigma_f\phi_0\psi_1\rangle + a_1\langle\psi_1^*\alpha\Sigma_f\phi_0\psi_1\rangle \\ & - a_1\langle\psi_1^*\Sigma_{ext}\psi_1\rangle - \langle\psi_1^*\delta\Sigma_c\phi_0\rangle \end{aligned} \tag{4.18}$$

where $\Delta\lambda_1 = \dfrac{\lambda_0 - \lambda_1}{\lambda_0\lambda_1}$ is the eigenvalue separation of the first harmonic mode,

Σ_{ext} = the uniform boron control cross-section,

and, $\delta\Sigma_c$ = the flux damping control cross section representing the partial length rods which are moved in an assymmetric fashion.

The additional macroscopic cross-section Σ_{ext} is introduced so that the total power output (gross power output) is maintained at a constant value.

The two terms on the left hand side of equation (4.17) are negligible compared to the terms on the right hand side, andtherefore they may be neglected. We may also assume that the flux comes into quick equilibrium with Xenon (few hours compared with the time-scale of Xenon oscillations which is in the range of few tens of hours). This decouples the modal equations. Equation (4.17) may now be written as,

$$a_1 = \frac{-b_1 \langle \psi_1^* \sigma_X \Sigma_f \phi_O \psi_1 \rangle - \langle \psi_1^* \delta\Sigma_c \phi_O \rangle}{(\Delta\lambda_1) \langle \psi_1^* \nu \Sigma_f \psi_1 \rangle - \langle \psi_1^* \alpha \Sigma_f \phi_O \psi_1 \rangle + \langle \psi_1^* \Sigma_{ext} \psi_1 \rangle}$$

$$\triangleq g_1 b_1 + u \qquad (4.19)$$

Putting eq.(4.18) into eq.(4.16), the following equation for Xenon and Iodine time-dependent coefficients will be obtained :

$$\begin{bmatrix} \dot{b}_1 \\ \dot{c}_1 \end{bmatrix} = \begin{bmatrix} g_2 & \lambda_I \\ g_1 \gamma_I & -\lambda_I \end{bmatrix} \begin{bmatrix} b_1 \\ c_1 \end{bmatrix} + \begin{bmatrix} g_3 \\ \gamma_I \end{bmatrix} \qquad (4.20)$$

where $g_2 = -\lambda_X - \frac{\nu}{\lambda_1} \langle \psi_1^* \sigma_X \Sigma_f \phi_O \psi_1 \rangle + g_1 g_3$

and, $g_3 = \gamma_X - \frac{\nu}{\lambda_I} \langle \psi_1^* \sigma_X X_O \psi_1 \rangle$

Equation (4.19) is of the form,

$$\dot{X} = AX + BU \tag{4.21}$$

which is the standard matrix form of the linear control problem. The control U is constrained to lie in the allowable range $R \geq u \geq -N$, which represents the positive and negative first harmonic worths of inserting and withdrawing the control bank from its normal position.

<u>Flux Tilt</u> : The following definition of flux tilt is often used in the literature :

$$\text{Tilt} = \frac{\langle\text{source density}\rangle_{left} - \langle\text{source density}\rangle_{right}}{\langle\text{total source density}\rangle} \tag{4.22}$$

Using eq.(4.4) in eq.(4.21),

$$\text{Tilt} = \frac{\langle\nu\Sigma_f(\phi_o + \delta\phi)\rangle_{left} - \langle\nu\Sigma_f(\phi_o + \delta\phi)\rangle_{right}}{\langle\nu\Sigma_f(\phi_o + \delta\phi)\rangle}$$

$$= \frac{\langle\nu\Sigma_f |a_1\psi_1|\rangle}{\langle\nu\Sigma_f\phi_o\rangle}$$

$$= \frac{a_1\lambda_1}{\lambda_o} \tag{4.23}$$

Eq.(4.18) represents the constraint relation between the flux tilt and perturbing reactivity. Using eq.(4.18) in eq.(4.22), we obtain the following expression for the flux tilt :

$$\text{Tilt} = (g_1 b_1 + u)\frac{\lambda_1}{\lambda_o} \tag{4.24}$$

<u>Optimal Control Formulation</u> : The optimal control problem may now be stated as follows : <u>For a given assymmetric Xenon and Iodine</u>

amplitudes $b_1(0)$ and $c_1(0)$, force both the time dependant Xenon and Iodine amplitudes $b_1(t)$ and $c_1(t)$ to zero and simultaneously minimize the performance criterion

$$J = \int_0^T dt \tag{4.25}$$

where T is the final control time. Solutions to the problem will therefore be *time optimal* since the control objectives are acheived in minimum amount of time.

Pontryagin's principles of optimal processes are now applied to obtain solutions to the above optimal control problem. The hamiltonian function H is obtained by appending the integrand of the performance criterion to the dot product of the state vector $\dot{\mathbf{X}}$ and the adjoint vector **P** resulting in the following expression :

$$\begin{aligned} H &= 1 + \mathbf{P} \cdot \dot{\mathbf{X}} \\ &= 1 + \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}^T \left\{ \begin{bmatrix} g_2 & \lambda_I \\ g_1\gamma_1 & -\lambda_I \end{bmatrix} \begin{bmatrix} b_1 \\ c_1 \end{bmatrix} + \begin{bmatrix} g_3 \\ \gamma_I \end{bmatrix} u \right\} \\ &= 1 + p_1(g_2 b_1 + \lambda_I c_I) + p_2(g_1\gamma_I b_1 - c_1\lambda_I) + S\,u \end{aligned} \tag{4.26}$$

where $S = p_1 g_3 + p_2\gamma_I$. Application of Pontryagin's theorems and associated theorems for minimization of H will lead to the following bang-bang control :

$$u = \begin{cases} +R & \text{if } S < 0 \\ -N & \text{if } S > 0 \end{cases}$$

The adjoint vector has to satisfy the following relation :

$$\dot{\mathbf{P}} = -\left(\frac{\partial H}{\partial X} \right)^T$$

or,

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} -g_2 & -g_1\gamma_I \\ \lambda_I & \lambda_I \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} \tag{4.27}$$

Eq. (4.27) may now be solved for P_1 and P_2 to obtain

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \frac{e^{-\alpha t}}{\omega} [A] \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} \tag{4.28}$$

where,

$$[A] \equiv \begin{bmatrix} \omega \cos\omega t - g_2 \sin\omega t + a \sin\omega t & - g_1\gamma_I \sin\omega t \\ - \lambda_I \sin\omega t & \omega\cos\omega t + \lambda_I \sin\omega t + a \sin\omega t \end{bmatrix}$$

$a = (g_2 - \lambda_I) / 2$, the growth factor,

$\omega = \frac{1}{2} \sqrt{-(\lambda_I + g_2)^2 - 4 \lambda_I\gamma_I g_1}$, the angular frequency

and p_{10} and p_{20} are final conditions of the adjoint variables.

Hence, S is of the form

$$S = Ce^{-\alpha t} \sin(\omega t + \psi) \tag{4.29}$$

where C and ψ are constants. From this equation, it is clear that, for $C \neq 0$, any one type of control cannot last for more than (π/ω) units of time.

The Xenon and Iodine imbalances (as opposed to the Xenon and Iodine concentrations in the shutdown problem considered earlier) form the axes of the phase plane. At any given time the reactor state can be be considered as a single point on this phase plane.

The distance of the operating point from the origin of the phase plane is a direct indication of the amount of deviation from desired reactor condition. The aim of the control strategy is therefore to return the reactor operator point on the phase plane to the origin in the minimum amount of time.

For an unstable reactor system, the solution to the uncontrolled system equations as seen in the XIPP is a divergent spiral which begins from the given initial imbalance condition. With control applied in the proper direction to oppose the build-up of the oscillation, the solution is again a divergent spiral, but one whose center is shifted from the origin. As seen in Fig.(4.2), which is drawn for the case of the symmetric control action, there is a specific controlled spiral in each half-plane labelled S_1 and S_2, which takes the initial condition along its path to the origin. We call this special spiral a *Switching Curve*, because optimal minimal control of a free Xenon oscillation will be acheived if we turn on the correct control action when divergent spiral for a free oscillation intersects the switching curve.

The solution of Eq.(4.20) for the symmetric Xenon and Iodine amplitude coefficients with control applied is

$$\begin{bmatrix} b_1(t) \\ c_1(t) \end{bmatrix} = e^{\alpha t} \, [B] \qquad (4.30)$$

where,

b_{10} and c_{10} are initial states of Xenon and Iodine, and

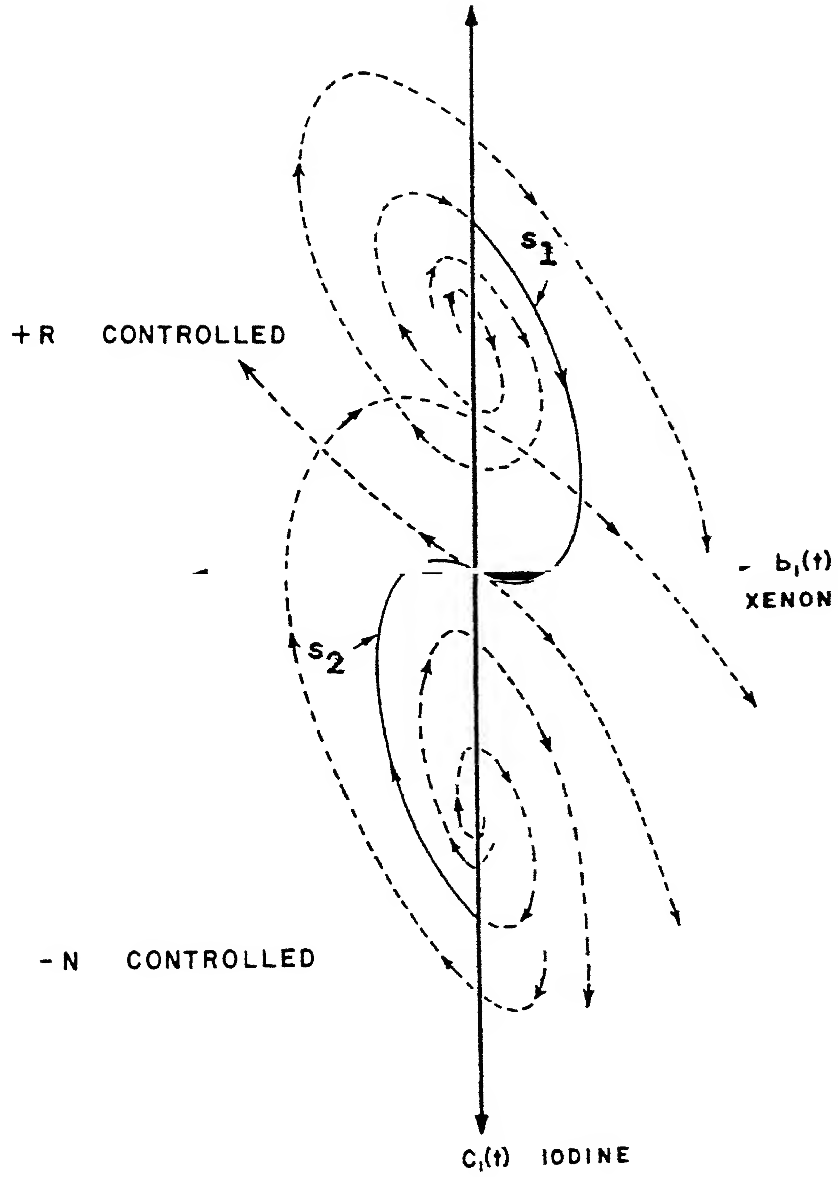


Fig. 4.2. CONTROLLED OSCILLATION SPIRALS [Ref.10].

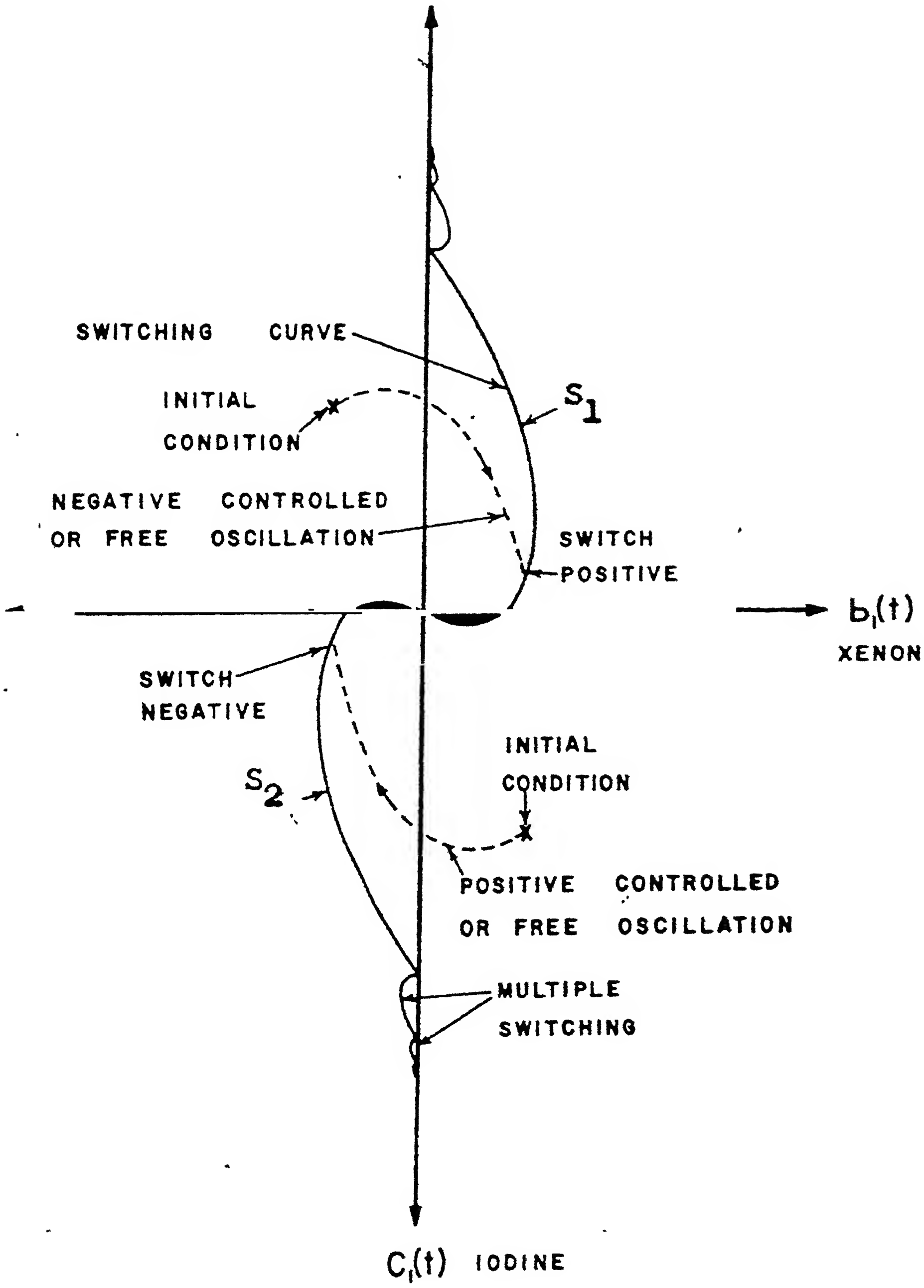


Fig. 4.3. SWITCHING CURVES [Ref.10].

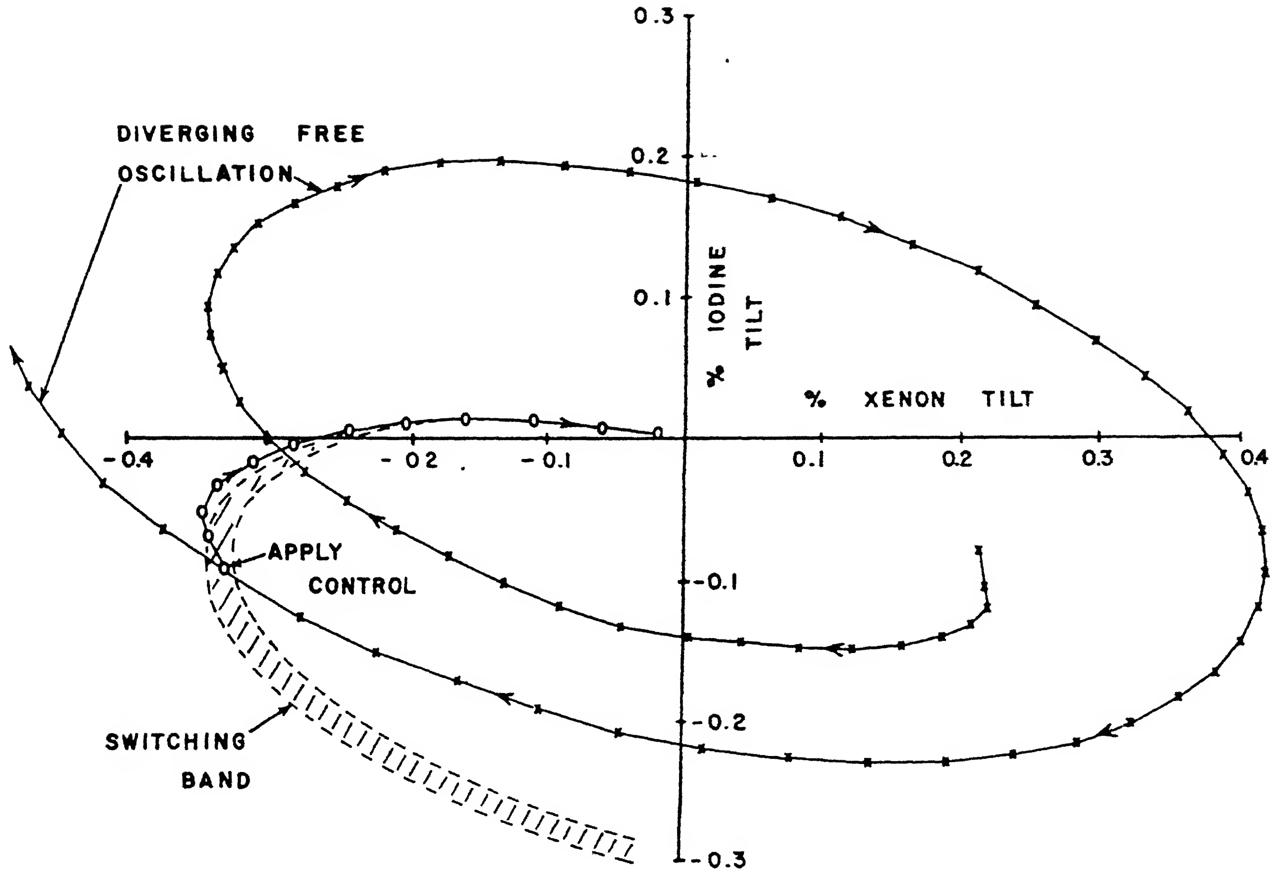


Fig. 4. 4. OPTIMAL CONTROL USING MEASURED PARAMETERS [Ref.10]

switching curves plotted, the first determined using the measured value of u and the second found using the calculated value of u, form a switching band. The growing spiral nature of the free oscillation is quiet obvious. In addition, when control is applied as predicted by the switching curve, the system response is seen to follow the switching curve quiet closely as oscillation is damped essentiallly to zero.

Fig.(4.5) shows an attempt to control the same model using a switching band curve determined from calculated values of g_1, g_2, and g_3. It is obvious by comparision with Fig.(4.4) that this switching curve implies switching should be initiated about 0.3 hour earlier than the previous case. The switching is in error due to the fact that the controlled response undershoots the origin and further control action is necessary. The curve is included here because it shows the large range of error that is tolerable. While the control action does not completely supress the oscillation, it significantly reduces it and places it in a region closer to the origin where larger errors can be tolerated. A very short subsequent control action completely subdues the oscillation less than half a period later.

The residual oscillations requiring secondary control action is larger in the case where control is applied too early. All control actions are initiated while the flux imbalance is increasing in absolute magnitude, and the direction of control results in a decrease in the absolute magnitude of the flux imbalance. All control terminates when flux imbalance without the control is determined to be zero. The zero point of flux tilt is

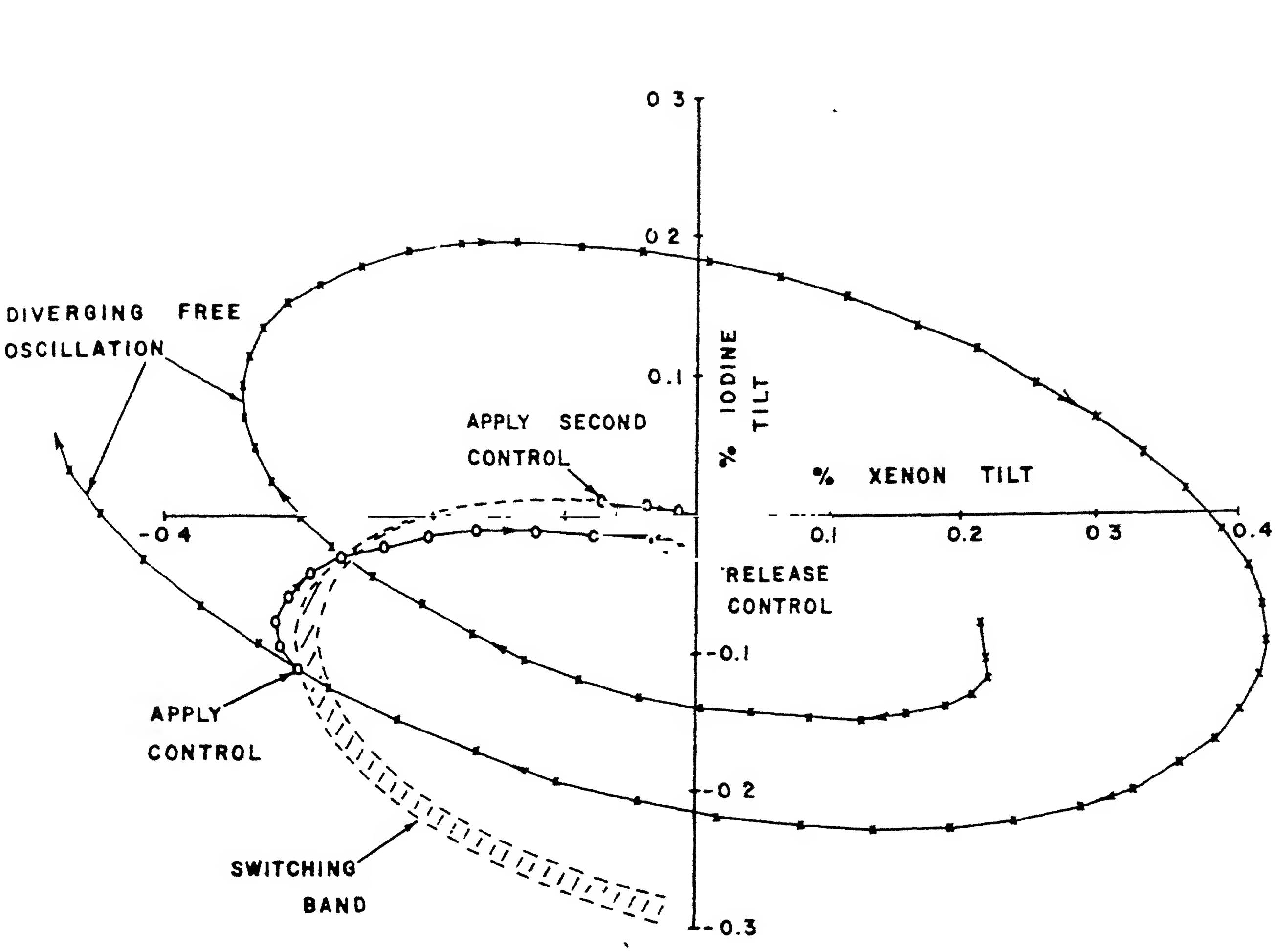


Fig. 4.5. OPTIMAL CONTROL USING CALCULATED PARAMETERS [Ref.10]

not necessarily a zero flux tilt. If the reactor is initially non-symmetric, due for example to the control rod configuration, the zero point is shifted to coincide with the normal non-symmetry. For these cases the definition of flux tilt is modified so that

Tilt = Actual tilt - Initial tilt.

The secondary action that is required when undershoot or overshoot occurs is now examined. Early switching leads to an undershoot. Normally, control would be released when Xenon tilt changes sign. It was seen that if this strategy is followed, the free oscillations will continue for another half period until the switching curve is again intersected. On the other hand, if the control is released before the Xenon tilt changes sign, the system response very quickly intersects the switching curve. This later action is not predicted optimally, but appears to be an excellent strategy.

Later switching tends to cause an overshoot. In this case, early release of control action will cause the response to intersect the switching curve approximately half a period later. It appears that a better strategy is to release the control action after the Xenon tilt changes sign, or not at all, which causes the response to intersect the switching curve soon thereafter.

4.6. Expert System Model For Control Of Spatial Oscillations

A model for expert system based optimal control has been discussed in Chapter-3. The expert system for the optimal control of Xenon induced spatial power oscillations will have same model

as for the shutdown case as shown in Fig.(3.3), except for the following functional differences.

(i) The function of the user block is the same as discussed before. However, the user input in this case will be the initial state of the perturbed reactor described in terms of Xenon and Iodine deviations from their normal values. This enables the initial reactor to be determined on the Xenon - Iodine Phase Plane.

(ii) The different sources of knowledge incorporated into the knowledge base are the numerical computation results obtained for the optimal control of spatial oscillations, typical patterns of the time-programs, and domain experts. The starting point in the rule-chaining procedure is to determine the reactor operating point as seen on the Xenon - Iodine Phase plane. The distance of the operating point from the origin indicates the size of the oscillation. From the knowledge of the case studies, it is clear that the control action to return the operating point to the origin can be initiated every (π/ω) units of time, where ω is the angular freqency of the oscillation. Since the reactor flux may be estimated directly from measurable process variables, the parameters g_1 and g_2 may be computed to yield the value of ω. From typical patterns of time programs, the time required for the operating point to reach the switching curve may be approximated. This time interval depends on the distance of the operating point from the curve. The switching curve for the reactor is fixed for a specified control rod strength. Thus the recommended control action for a given operating point consists of time of free

oscillations, the switching-time which takes place when the free oscillation trajectory intersects the switching curve, and the time required for the operating point to move along the switching curve to the origin. The switching curve is illustrated for an example in Fig.(4.4).

(iii) The reactor model describing the dynamics of spatial oscillations is incorporated into the solution analyzer. At each control time step the model predicts the state and control vectors at the next time step. If it has been predicted that the trajectory crosses the switching curve in the next time interval, the control action is initiated and the reactor state at the end of the control time-step is estimated. If the reactor state has been returned to the origin, the current candidate control vector is executed. Otherwise, the time at which the control action is initiated is modified and the process is repeated until the correct control-time is obtained.

4.7. Conclusions

In this chapter, optimal control solutions for damping one-dimensional Xenon induced spatial power oscillations in thermal nuclear power reactors were obtained using the lambda mode expansion method. A case study with a numerical example was discussed. An expert system for controlling the spatial oscillations was proposed, which was similar to the one discussed in chapter-3 for the case of shutdown control. From these studies, the following may be concluded :

(i) A time-optimal bang-bang type control can be obtained

for suppressing Xenon-induced spatial power oscillations in a thermal nuclear power reactor using the theory of optimal processes as developd by Pontryagin *et al*.

(ii) Sub-optimal control strategy employing a secondary control action may be required if the optimal control trajectory undershoots or overshoots the origin of the Xenon-Iodine Phase Plane resulting from the approximations involved in the formulation of the problem.

(iii) An expert system can be developed for the optimal control of spatial oscillations in a nuclear reactor using the knowledge obtained from case studies and other operational knowledge about the reactor system. Such a system will be superior compared to a solution based on analytical approach due to its ability to include heuristic knowledge.

CHAPTER 5

CONCLUSIONS

The two major problems of poisoning-out and spatial power oscillations associated with the presence of Xenon-135 poison in thermal nuclear power reactors were discussed. Analytical solutions to both of these problems were obtained by application of theorems of optimal processes as developed by pontryagin *et al.* The concept of Xenon - Iodine Phase Plane was used in both cases to define the optimal control laws. For the full-shutdown case, digital simulation results were obtained and used in the rules contained in knowledge base of the expert system. Some numerical examples for spatial oscillations problem were discussed and an expert system similar to the one discussed for the full-shutdown case was proposed. PWR data was used in case studies.

The major findings from the work done in the thesis may be summarized as follows :

(i) For a given reactor override capability a reactor operating at low power level may be shutdown without applying control to prevent poisoning-out. There is a threshold, only below which such immediate shutdown can be allowed to take place. In the example considered, this threshold was found to be 0.96×10^{14} n cm^{-2} s^{-1}.

(ii) Reactors normally require a one-pulse control during shutdown process to avoid poisoning-out. The range of one-pulse control encompasses operating points that are most commonly encountered in practice. The one-pluse range was found to be 0.96 $\times 10^{14}$ to 0.56×10^{17} n cm^{-2} s^{-1} for the example considered in the thesis.

(iii) For operating points not acessible even for one-pulse control, a seqeuntial operation at zero power, variable (decreasing) power, and finally at full power is required.

(iv) Xenon induced spatial power oscillations can be suppressed in a time-optimal manner by applying a suitable bang-bang type control. A prerequisite is a precise estimate of reactor flux tilt which enables the determination of reactor operating point on the Xenon - Iodine Phase plane.

(v) Due to the approximations involved in formulating the spatial oscillations problem, the solutions that are generated may not be truly optimal. Sub-optimal control strategies may be readily generated in such a situation.

(vi) It is possible to develop expert systems for optimal control of poisoning-out and spatial oscillations from the numerical computation results. The control obtained will be superior compared to conventional methods since a practically

near-optimal control strategy may be evolved using heuristics and other practical knowledge of the system.

The scope for future work may be pointed out as follows :

(i) Temperature feedback and Samarium-149 poison effects may also be included in the consideration of optimal shutdown control problem.

(ii) An expert system for actual control of a reactor can be tested for determining the effectiveness of the method proposed.

(iii) Implementation of the analytical results for control of spatial oscillations may be carried out for an actual reactor. Numerical results from such a case study can be used to develop the proposed expert system.

REFERENCES

1. Z.R.Rosztoczy and L.E.Weaver, " Optimum Reactor Shutdown Program for Minimum Xenon Build-up ", Nuclear Science And Engineering, Vol.20, pp.318-323, (1964).

2. J.J. Roberts and H.P. Smith Jr.," Time Optimal Solution To The Reactivity-Shutdown Problem", Nuclear Science And Engineering, Vol.22, pp.470-478, (1965).

3.M. Ash, "Application of Dynamic Programming To Optimal Shutdown Control ", Nuclear Science And Engineering, Vol.24, pp.77-86, (1966).

4.M.Ash, " Optimal Shutdown Control Of Nuclear Reactors ", Academic Press, New York, (1966).

5.A.M. Christie and A.M.Poncelet, " On Control Of Spatial Xenon Oscillations ", Nuclear Science And Engineering, Vol.51, pp.10-24, (1973).

6. D.M. Wiberg, "Optimal Feedback Control Of Spatial Xenon

Oscillations In Nuclear Reactors", PhD thesis, California Institute Of Technology, (1965).

7.A.A.Bassioni and C.G.Poncelet, "Minimal Time Control Of Spatial Xenon Oscillations In Nuclear Power Reactors", Nuclear Science And Engineering", Vol.54, pp.166-176, (1976).

8. H.Ukai, Y.Morita, Y.Yada and T.Iwazumi, " Control Of Xenon Spatial Oscillations During Load Follow Of Nuclear Reactor Via Robust Servo Systems ", Journal Of Nuclear Science And Engineering, Vol.26, pp.237-253, (1966).

9.J.Canosa And H.Brooks, " Xenon-Induced Oscillations ", Nuclear Science And Engineering, Vol.26, pp.237-253, (1966).

10. A.T. Chiang, W.D. Beckner, R.A.Rydin, " A Reformulation Of The Xenon Optimal Control Problem Using Lambda Modes", Proceedings Of The Conference On Computational Methods In Nuclear Engineering, April 15-17, 1975, Charleston, South Carolina.

11. Y.Shinohara, "Application Of An AI Method To Optimal Reactor Control Problems", Proceedings Of The International Topical Meeting On Artificial Intelligence And Other Innovative Computer Applications In Nuclear Industry, Snowbird, Utah, Aug31-Sep2, 1987, Plenum Press, New York,(1988).

12. D.Majumdar, C.Majumdar, J.I.Sackett, Proceedings Of the International Topical Meeting On Artificial Intelligence And Other Innovative Computer Applications In The Nuclear Industry, Snowbird, Utah, Aug31-Sep2, 1987, Plenum Press, New York, (1988).

13. R.E.Uhrig, "Expert Systems And Their Use In Nuclear Power Plants", Advances In Nuclear Science And Technology, Vol.21, Edited by J.Lewins and M.Becker, Plenum Press, New York, 1990.

14. Special Issue On Artificial Intelligence Applications In Nuclear Power, Prepared by D.Majumdar, Nuclear Engineering And Design, Vol.113, No.2, April (II), (1989).

15. L.S.Pontryagin, V.G.Boltyanskii, R.G.Gamkrelidze and E.F.Mishchenko, "The Mathematical Theory Of Optimal Processes", Edited by L.W.Neustadt, Interscience Publishers, New York, (1963).

16. R.Bellman, "Dynamic Programming", Princeton University Press, Princeton, N.J., (1957).

17. A.F.Henry, "Nuclear-Reactor Analysis", The MIT Press, (1975).

18. W.M.Stacey Jr., "Modal Approximations : Theory And An Application To Reactor Physics", Research Monograph No.41, The MIT Press, (1967).

19. J.A. Bernard, "Application Of Artificial Intelligence To Reactor And Plant Control", Nuclear Engineering And Design, Vol.113, pp.219-227, (1989).

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